

THE GALACTIC DISTRIBUTION OF CARBON MONOXIDE:

AN OUT-OF-PLANE SURVEY

Richard Stuart Cohen

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ABSTRACT

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Galactic CO line emission at 115 GHz has been surveyed in the region $15^\circ < \ell < 60^\circ$ and $-1.5^\circ \leq b \leq 1.5^\circ$ using the Columbia 4-foot telescope (HPBW 8 arc minutes).

This survey, which comprises the first systematic out-of-plane data on CO emission from the Galaxy, confirms the finding of previous in-plane studies that CO is concentrated in a ring 6 kpc in radius. It provides the first determination of the thickness of this molecular ring as a function of galactic radius and shows that CO is displaced from the conventional galactic plane. These results were arrived at by least-squares fitting the survey data to a circularly symmetric model of the Galaxy. The average half-thickness at half maximum of the molecular ring is 59 pc and the average displacement of the ring with respect to the $b=0^\circ$ plane is -40 pc.

A comparison of the CO and HI distributions shows that there are marked differences in the distributions of these species, both radially and out of the plane. The CO distribution is similar to those of HII regions, supernova remnants, and other young galactic objects. It is argued that CO is probably the best available tracer of this component of the Galaxy.

A detailed discussion of the antenna characteristics, including the radiation pattern and pointing characteristics is presented.

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I. INTRODUCTION

Since the discovery of widespread interstellar molecules in the late 1960's, it has become clear that molecular clouds are an exceedingly important constituent of the interstellar gas. They comprise a large fraction, probably over half, of the interstellar mass and are the raw material from which stars evolve. So far, most of the effort in molecular radio astronomy has been directed toward the study of well-known objects such as HII regions, stars, and nearby dark clouds. Recently investigation has begun into the distribution of molecular clouds on a galactic scale. The initial steps in this investigation are naturally to determine several of the overall characteristics of the molecular matter: its distribution both as a function of galactic radius and of distance from the galactic plane; and the nature of its fine scale structure, for example its clumpiness and velocity dispersion.

The best tool currently available for studying these problems is line emission by carbon monoxide. Although H_2 is by far the most abundant molecule, it has no dipole moment and cannot be seen at radio frequencies. It can be seen from satellites in ultraviolet absorption, but these observations are generally limited by interstellar extinction to within a distance of about 2 kpc from the earth. On the other hand,

emission from the first rotational transition of CO at 115 GHz has been found to be easily detectable in virtually all massive molecular objects. Although this line is usually found in clouds in which it is optically thick, these clouds seem to be distributed sparsely enough that there is no serious problem of obscuration of distant clouds by foreground matter. Thus, using CO as a probe, molecular matter can be detected throughout the Galaxy.

This thesis reports the results of the first systematic out-of-plane survey of CO in the galactic disk. We will obtain the distribution of CO -- and by implication the distribution of molecular matter generally -- both as a function of galactic radius and, in an important new result, as a function of distance from the galactic plane.

Radial distribution of molecules

Until now, the distribution of molecular matter had been systematically investigated only in the galactic plane. Two independent groups have made surveys of CO in the plane (Scoville and Solomon 1975; and Burton et al. 1975, Burton and Gordon 1976) and both have arrived at similar conclusions: excluding the galactic center region ($R < 600$ pc), where intense emission is seen, the bulk of the CO emission is concentrated in a ring about 2 kpc wide and centered at about $R = 5.5$ kpc from the galactic center. This CO

distribution is similar to those of HII regions (Burton et al., 1975), diffuse HII gas (Gordon and Cato, 1972), supernova remnants (Ilovaisky and Lequeux, 1972; Kodaira, 1974), and pulsars (Seiradakis, 1977), i.e. all the Population I objects whose galactic distributions are known with the exception of HI. The HI distribution is however very different: HI emission rises smoothly from near the galactic center to a peak at $R = 14$ kpc.

There is also a striking correlation between CO emission and γ -ray emission (Puget and Stecker, 1974; Paul et al., 1976). Because high energy γ -rays (> 100 MeV) are produced by the interaction of cosmic rays with the nuclei of interstellar gas (Stecker, 1977), the γ -ray emissivity is expected to be proportional to the product of the interstellar mass density and the cosmic ray density. Thus the correlation of γ -ray emission with molecular emission, rather than with 21 cm emission, suggests that H_2 is the dominant component of the interstellar mass.

It should be emphasized that before the present work the galactic CO measurements were made at $b = 0^\circ$, while the data for the other tracers is integrated across the disk of the Galaxy. A proper comparison therefore requires extensive out-of-plane CO data.

Molecular Mass of the Galaxy

To estimate the total molecular mass of the Galaxy from the in-plane CO surveys, it has been assumed that the average ratio of H_2 mass to CO luminosity for molecular clouds is the same throughout the Galaxy. In addition, as the CO surveys have been confined almost exclusively to the galactic plane, a thickness of 120 pc -- estimated from a few out-of-plane measurements -- was used for the entire galactic disk.

Several methods have been used to determine the mass-to-luminosity ratio for molecular clouds. With a simple radiative transfer model for molecular clouds, Burton and Gordon (1976) used measurements of the intensities of the $J = 1 \rightarrow 0$ transitions of both ^{13}CO and ^{12}CO to calculate the column density of CO. Then, taking the value of $[CO]/[H_2]$ from local measurements to be constant over the entire Galaxy, they obtained the H_2 column density. Alternatively, the actual ratio of H_2 column density to CO intensity in the direction of the galactic center can be measured (Stecker et al. 1975). The total hydrogen column density -- molecular and atomic -- can be obtained from either infra-red extinction or X-ray absorption data; the atomic component, as

measured at 21 cm, must then be subtracted to get the H_2 column density. Each of these methods gives about the same total galactic molecular mass: $2 \times 10^9 M_\odot$. Unfortunately, they all involve so many uncertain assumptions that they may not give more than order-of-magnitude estimates.

In principle, the mass can also be obtained directly from the γ -ray data. The γ -ray emission is proportional to the cosmic ray density times the total gas density. Thus if a model cosmic ray distribution is assumed and if the distribution of atomic matter is known, then the molecular mass can be computed. Models with moderate (less than a factor of two) radial variation in the cosmic ray density give masses consistent with those derived from CO data (Stecker et al., 1975). However, the cosmic ray density is poorly known at large distances. If the hydrogen density could be more accurately determined from the radio data alone, the γ -ray data could be used more effectively as a measure of the cosmic ray density in the interiors of molecular clouds.

Summarizing, the best current value for the H_2 mass of the Galaxy is $2 \times 10^9 M_\odot$; however, this is really little more than an order-of-magnitude estimate. When

detailed studies of distant molecular clouds have been made, it will be possible to better determine their molecular content and improve the accuracy of the mass estimate.

Thickness of the Galactic Plane

As we have seen, to determine the H_2 distribution it is essential to know the thickness of the molecular layer. In addition, the thickness is interesting in its own right as a probe of galactic structure. Investigations of the thickness of the stellar component of the Galaxy date from the beginnings of Galactic astronomy (see, e.g. Oort, 1932). Two major results of these studies have been of great importance. First, from measurements of the density and velocity dispersion of a single type of star as functions of height above the galactic plane, z , it has been possible to determine the gravitational field and thus the total mass density, both as functions of z (Oort, 1965; Woolley, 1965); and second, information on the scale heights of the various stellar types has been crucial in classifying the stars into populations (Blaauw, 1965).

In spite of the great success of these stellar programs, they do suffer from a severe limitation: optical observations can generally be made only within about two

kiloparsecs from the sun. This restriction can be removed if radio data is used. We will briefly summarize the radio data available on the non-local thickness of the galactic disk. The radio observations can be divided into two broad categories: diffuse gas and discrete objects. The HII gas can be detected by hydrogen recombination line observations. (Actually there is some question as to whether these lines really come from diffuse gas or simply from weak HII regions.) Lockman (1976) and Gordon et al. (1972) have measured the half thickness at half maximum ($z_{1/2}$) of the HII gas at between 40 and 80 pc. However, these measurements are extremely time consuming and this result is based on a total of only 10 spectra. The distribution of HI gas, easily observed at 21 cm, has been thoroughly studied, but far from the sun it is very difficult to distinguish the diffuse HI gas from the HI clouds. Using a rather complex stochastic model of HI clouds distributed in a diffuse medium, Baker and Burton (1975) obtained $z_{1/2} = 140$ pc for the diffuse gas. In addition to the observational problems, there are difficult problems in the interpretation of the thickness data because the dynamics of the diffuse gas are strongly influenced by thermal, cosmic-ray and magnetic pressures.

The effects of these pressures are reduced or eliminated

in the case of discrete objects. The discrete radio objects that can be seen at large distances include cold HI clouds, HII regions, supernova remnants (SNR's), pulsars, and molecular clouds. There is an enormous amount of HI data, but, as we have seen, to distinguish the clouds from the diffuse medium is extremely difficult. For $R < 10$ kpc, the Baker and Burton (1975) stochastic model gives an average value of $z_{1/2} = 95$ pc but no information on the variation of $z_{1/2}$ with R .

Pulsars, SNR's and HII regions can be observed throughout the Galaxy, but the number of objects bright enough to be seen up to 10 kpc away is less than 100 in each case. This is large enough to measure the average scale height of the Galaxy, but not adequate to measure the scale height as a function of galactic radius. For pulsars, $z_{1/2} = 330$ pc (Seiradakis, 1977), much larger than for other population I objects. The large thickness is presumably the result of large velocities given to the pulsars at their formation. For SNR's (Ilovaisky and Lequeux, 1972) and HII regions (Burton et al., 1975) $z_{1/2} \approx 67$ and 40 pc, respectively, consistent with other extreme population I objects. Ilovaisky and Lequeux did make a rough determination of the radial dependence of the thickness of SNR's and found that $\langle z^2 \rangle^{1/2}$ increases from 46 pc for $4 < R < 8$ kpc to 89 pc for $8 < R < 10$ kpc. If the thickness of

either HII regions or SNR's is to be determined in more detail, many new objects will have to be detected. It is not likely that this will be possible in the near future.

In contrast to the other radio objects we have considered, molecular clouds are extremely numerous. There are at least 10,000 according to the cloud density estimate of Scoville and Solomon (1975), and virtually all of these can be observed with existing telescopes. The molecular clouds are probably sufficiently heavy that, like stars, their motion is determined by gravity alone and not by the pressure of the interstellar medium (Mouschovias, 1975). If the thickness and velocity dispersion of the molecular clouds can be determined throughout the Galaxy, it may be possible to use them as a probe of the gravitational field on a galactic scale, just as stars have been used locally.

Prior to the present work, the best published measurement of the thickness of the CO layer had been made by Burton and Gordon (1976) based on a single series of spectra taken at $l = 21^\circ$ between $b = -2^\circ$ and $+2^\circ$. By generating a number of synthetic velocity vs latitude contour maps of CO emission based on models with various thicknesses of the CO layer and then choosing the map that best matched the actual observations, they estimated that the data were best fit with a Gaussian dispersion about the galactic plane of 50 pc. They

made no attempt to determine the variation of the thickness with galactic radius.

There are several reasons for a more detailed determination of the thickness of the molecular layer. First, the molecules are a major component of the interstellar medium, and a thorough investigation of their distribution is essential. Second, because of their large numbers and ease of detection, molecular clouds are the best tracer of the extreme population I component of the Galaxy. And third, because they move as pressure free objects, the molecular clouds may serve as a probe of the galactic gravitational field.

Present Work

To make an extensive survey of the molecular content of the Galaxy, the construction of a 4-foot radio telescope on the roof of the Pupin Physics Laboratory at Columbia University in New York City was undertaken in 1972. Because of the short wavelength at which it is operated -- 2.6 mm, the first rotational transition of CO -- the telescope, while quite small, has approximately the same resolution as the best that is available in any large scale 21 cm survey. The telescope was completed in December 1974, and a systematic survey of CO in the galactic disk was begun in November 1975. The results of the first phase of that survey are the subject of this dissertation.

II. APPARATUS

The observations were made using the 4-foot telescope at Columbia University in New York City. The telescope system, diagrammed in Figure II-1, consisted of five main elements: 1) the antenna, 2) the mount, 3) the front end which amplified the incoming microwave signal and converted it to a lower frequency, 4) the spectrometer or backend, and 5) the computer system which controlled the telescope and processed the data. Each of these elements will be described briefly below. More detailed descriptions can be found in the theses of those principally responsible for each element: details of the antenna characteristics and the mount alignment are in appendices to this thesis, the front end is described by H.I. Cong (1977), and the backend and computer systems are described by G. Chin (1977).

The antenna, built by Philco Ford and located on the roof of the Pupin Physics Laboratories, is a Cassegrain reflector with an 8 arc min beamwidth (FWHM) at the CO frequency. It consists of a 4-foot (1.2 meter) main reflector with $f/D = 0.375$ and a 6 inch (15 cm) secondary, giving an effective system $f/D = 2.8$. The antenna was specified and was mechanically measured by the manufacturer to have a surface accuracy of better than $\lambda/75$ at the 2.6 mm CO wavelength, assuring that it would be diffraction limited.

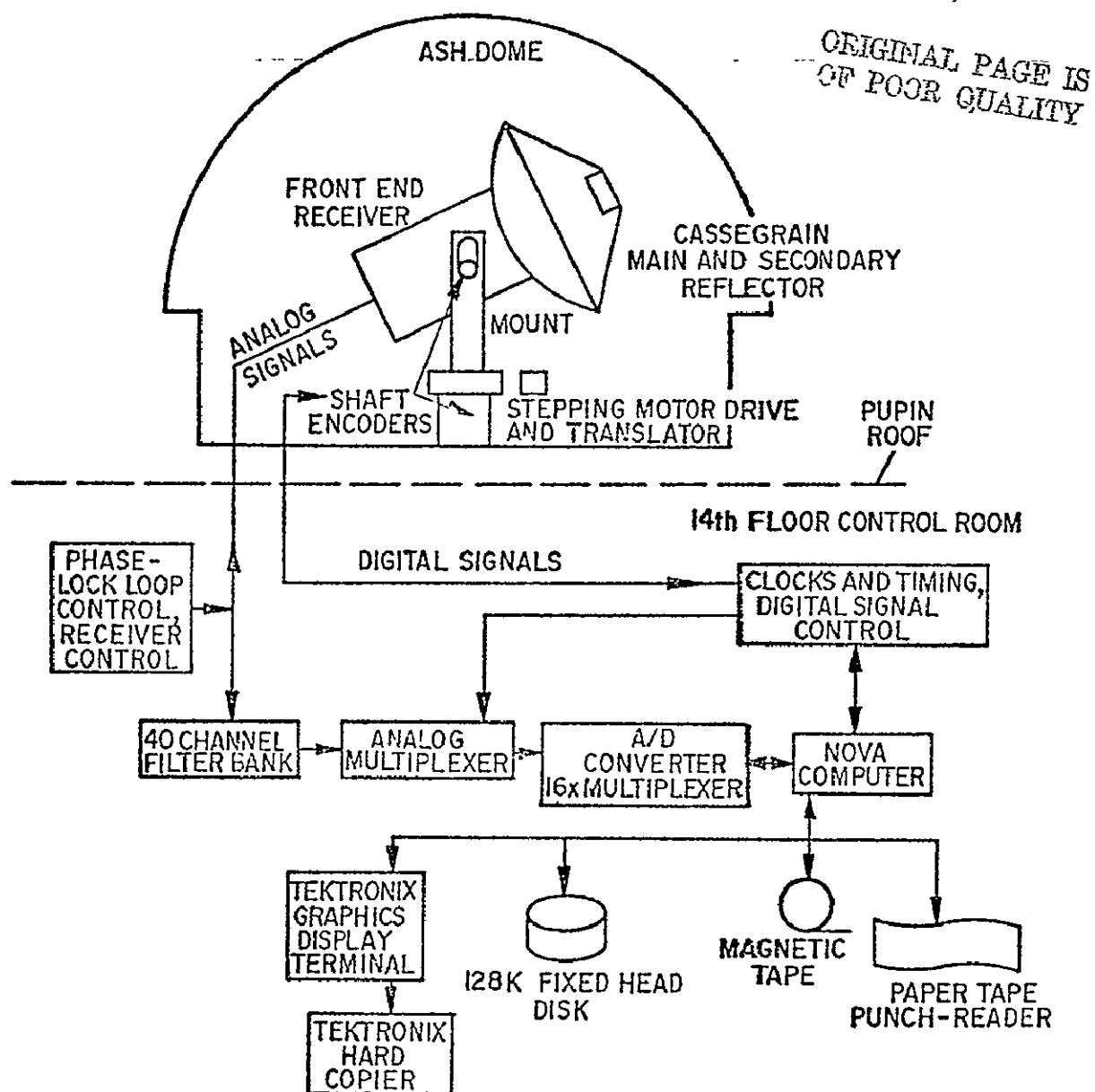


Figure II-1. Telescope block diagram.

To verify that these specifications were actually met and that the antenna was accurately aligned after delivery, the antenna pattern was measured at the CO frequency of 115 GHz using a signal transmitted from a distance of 420 meters. For comparison, a theoretical antenna pattern was calculated using scalar Kirchhoff diffraction theory. When the effects of the aperture blockage by the secondary mirror and its support structure were included, there was excellent agreement between the measured and calculated patterns (Figs. II-2 and II-3). These results were adequate to demonstrate that the dish did in fact meet its manufacturing specifications.

The effects of errors in feed horn positioning were also computed. It was found that the horn must be positioned to within 2 cm of the focus along the telescope axis and to within 11 cm transverse to the axis, and that the horn and telescope axes must be colimated to within 1°. These calculations were checked experimentally by deliberate maladjustment of the horn. In practice, the specifications were easily met. A more detailed discussion of all the calculations and measurements is given in appendix B.

The antenna was mounted on an altitude-azimuth drive built by the Physics department machine shop at Columbia

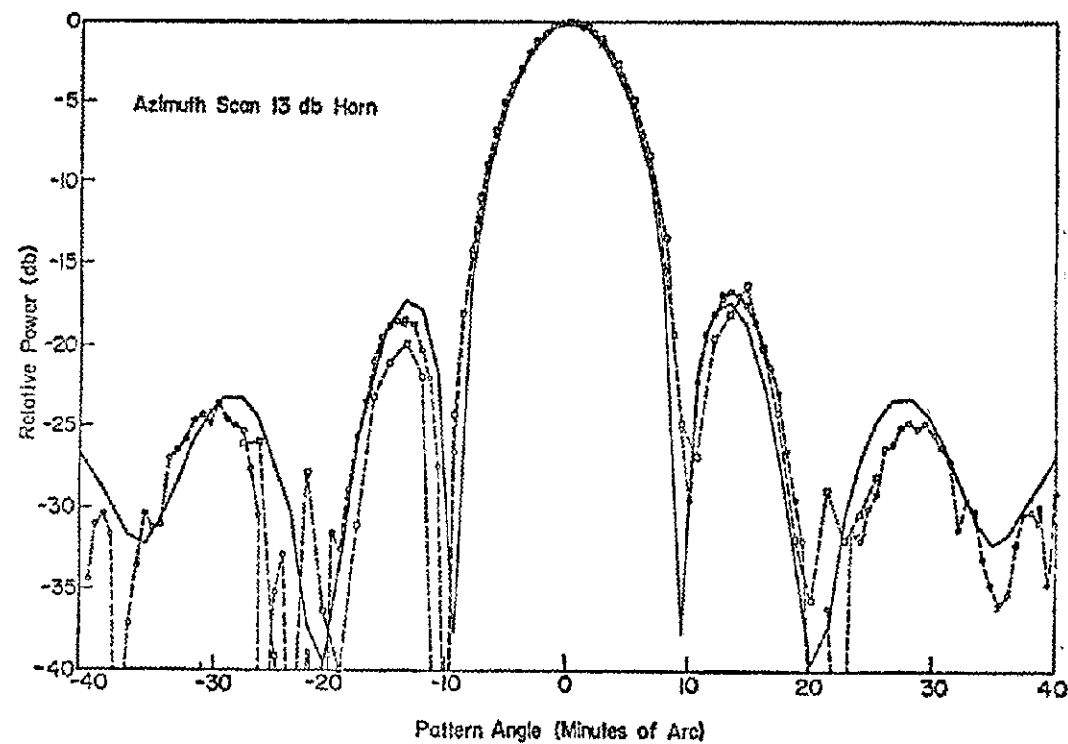


Figure II-2. Antenna pattern in horizontal plane with 13 dB edge illumination. Solid lines are the calculated patterns; dotted lines are the measurements.

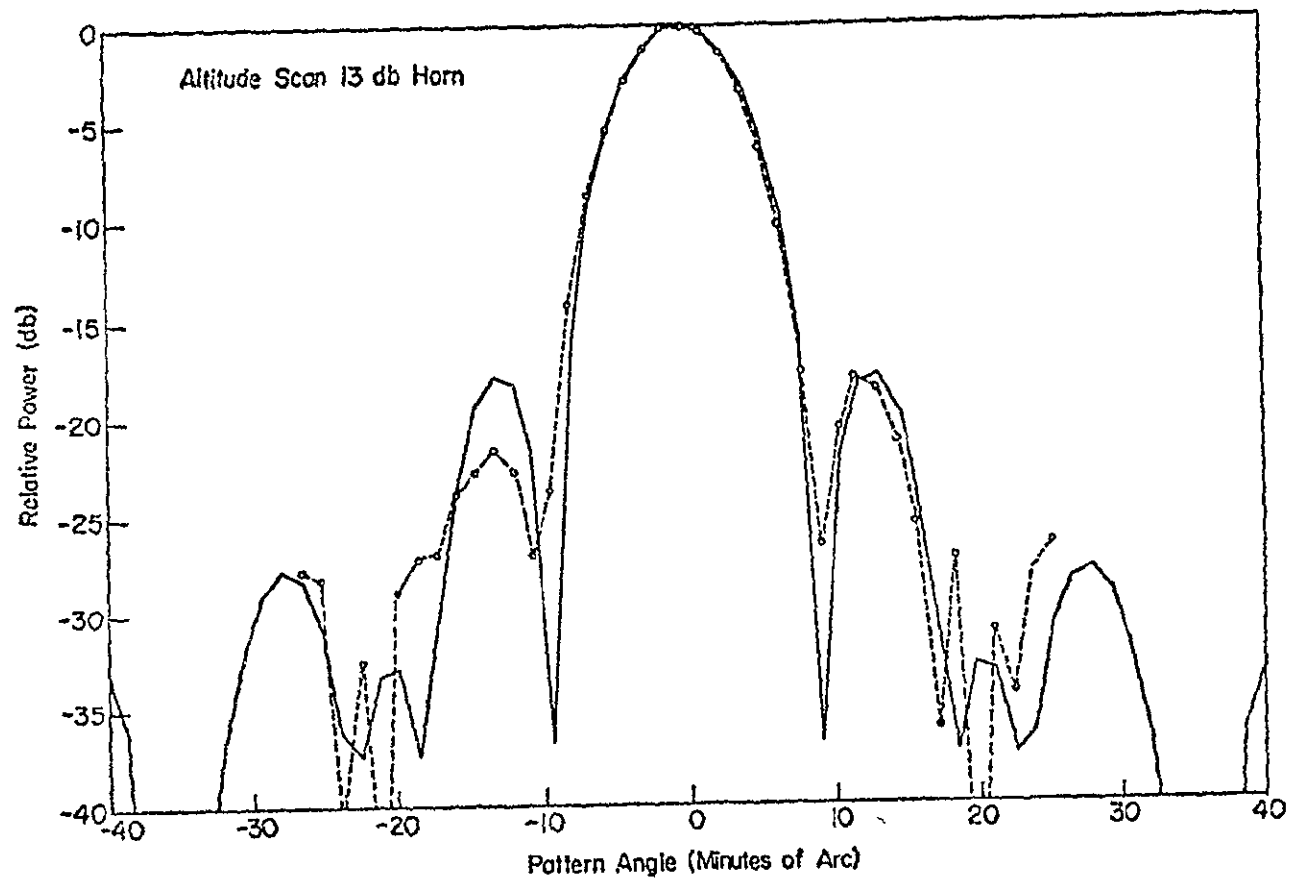


Figure II-3. Antenna pattern in vertical plane with 13 dB edge illumination. Solid lines are the calculated patterns; dotted lines are the measurements.

University. The position of the telescope was sensed by the computer system by means of 15 bit optical shaft encoders mounted on the telescope axes. Twenty times per second the computer read the encoder values and compared them with the desired values. If a correction was needed, the computer sent pulses to the stepping motors that controlled the telescope motion.

Because the sun and moon are the only astronomical sources whose positions can be readily measured at the CO frequency with a 4-foot telescope, the accuracy of the drive system could not be established over the entire sky by simple radio checks. Therefore, an optical telescope mounted on the radio dish and collimated with the radio axis of the telescope was used to check the pointing. The collimation was accomplished to within 0.25 arc minute by tracking the center of the sun with the radio axis and adjusting the optical telescope until its crosshairs were centered on the sun. Then, using the optical telescope, the pointing accuracy was measured against the positions of bright stars.

In the initial star-pointing tests, corrections were included for atmospheric refraction, deviation of the azimuth axis from the true vertical, non-perpendicularity

of the altitude and azimuth axes, and non-perpendicularity of the optical and altitude axes. The tests established that the two non-perpendicularity corrections were small enough they they could be dropped with no effect on the pointing accuracy. The magnitudes of the other corrections were determined from preliminary tests, and then a final check was made.

Figures II-4 and II-5 are plots of the residual errors for the 31 stars included in the final test. As these graphs show, it was possible to point the telescope to within 2 arc minutes of any desired position. The RMS error was 1.1 arc minutes. We estimate that the error was primarily due to three effects of about equal magnitudes: 1) quantization errors in the encoders and the stepping motor drive, 2) irregularities in the encoders, and 3) lack of rigidity in the mount. While the 2 arc minute ($\frac{1}{2}$ beamwidth) accuracy is adequate for present purposes, significant improvements are planned for the near future.

The pointing accuracy was confirmed by direct radio checks against the position of the sun that were made several times a week during the course of the observations. The accuracy of the telescope was further corroborated when

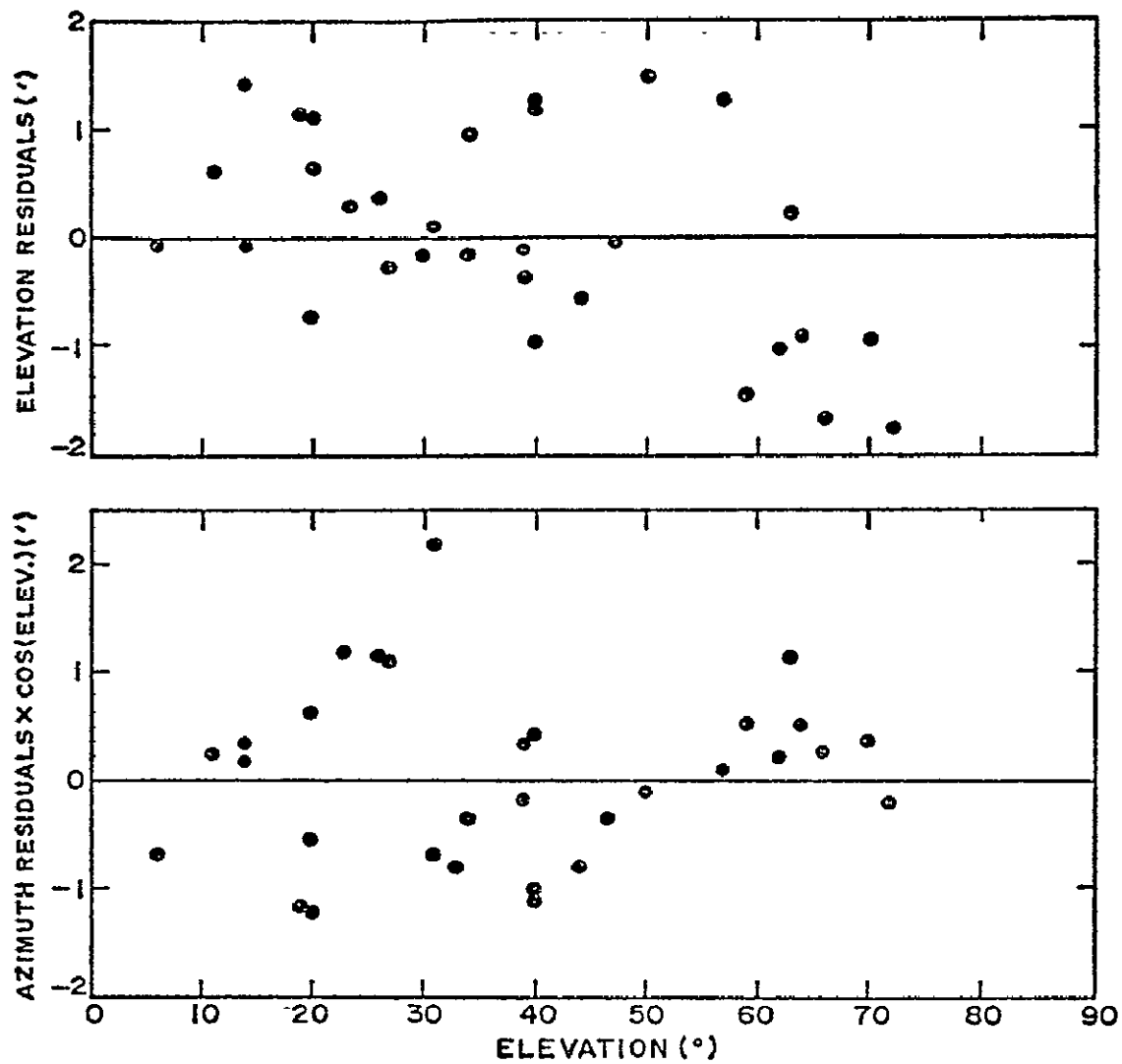


Figure II-4. Residual pointing errors; altitude axis.

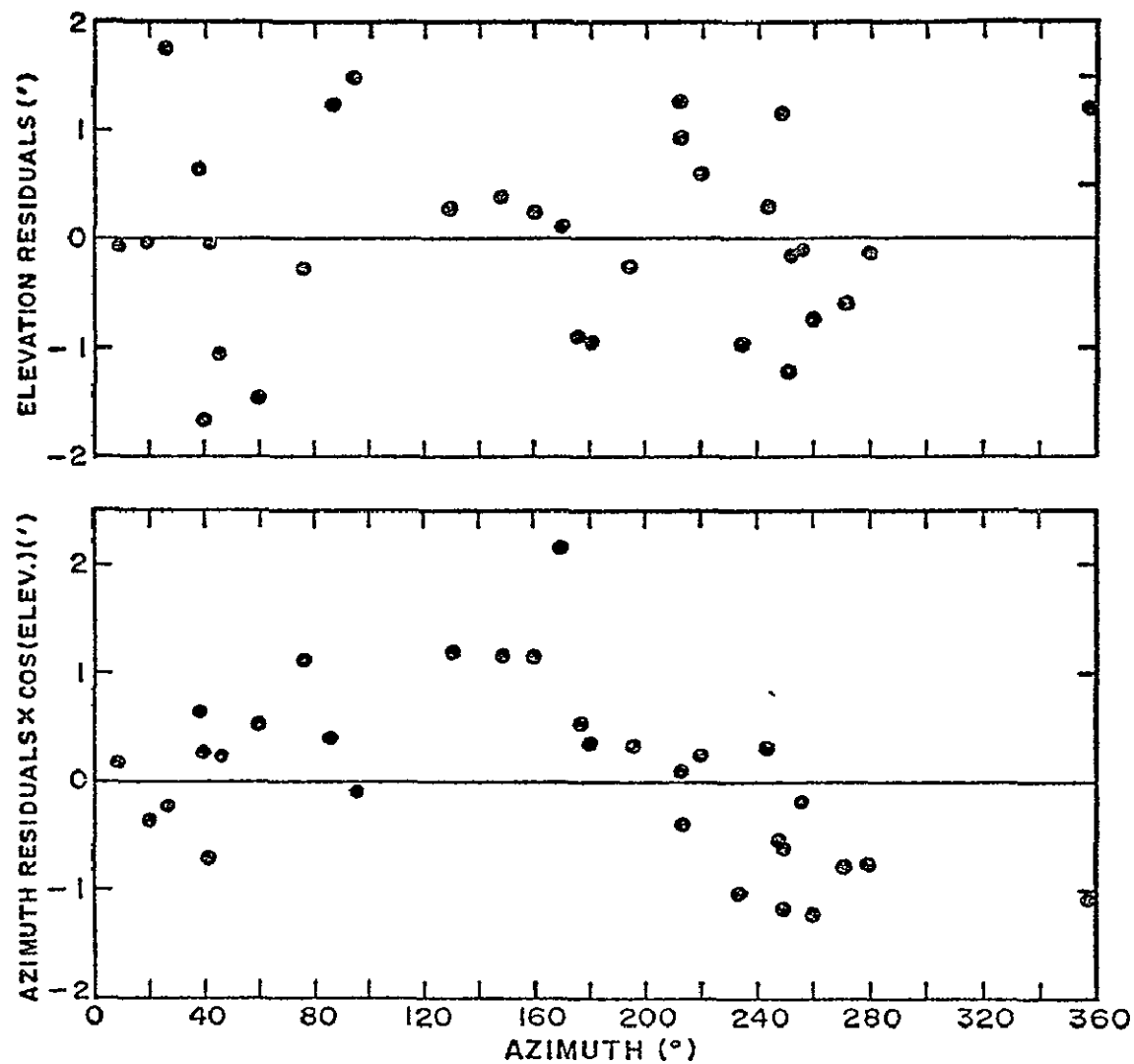


Figure II-5. Residual pointing errors; azimuth axis.

many sources mapped with it were later mapped with higher resolution telescopes. No discrepancies in position were ever found. Additional details of the telescope alignment procedure and the pointing tests are given in appendix A.

The front end was a double sideband superheterodyne receiver constructed by H.I. Cong. The incoming radiation was collected by a feed horn, and using a resonant ring injection cavity and a Schottky barrier mixer built by A.R. Kerr, was mixed with the signal from a phase locked klystron local oscillator to produce a 1.4 GHz intermediate frequency signal. This signal was amplified by a Micro-mega room temperature parametric amplifier with a 50 K noise temperature, further amplified by a low noise transistor amplifier, and then down converted to the second intermediate frequency of 150 MHz. (For details of the front end see Cong, 1977.) The complete system had a measured single sideband noise temperature of 1400 K referenced to the feed horn. The 150 MHz signal was amplified and sent to the spectrometer located in the control room on the floor below.

The spectrometer contained 40 three stage Butterworth filters, each with 1 MHz resolution full width at the half power points (corresponding to a Doppler velocity

resolution of 2.6 km sec^{-1} at the CO frequency), giving a total bandwidth of 40 MHz (104 km sec^{-1}). The outputs of the filters were detected and sent to analog integrators. Every 50 msec the outputs of the integrators were digitized by a 12 bit analog-to-digital converter and sent to the computer for further processing.

The computer system had two main functions: 1) to control the position of the antenna, and 2) to collect, process and store the data from the spectrometer. It consisted of a Data General Nova 1220 computer, a fast fixed head disk, 2 IBM compatible tape recorders, a paper tape reader and punch, a Tektronix graphic display terminal and hard copy unit, and a clock unit that provided time-of-day and synchronization signals. The programming was done by G. Chin and G. Tomasevich with assistance from the author. Details of the programs and hardware are given in Chin (1977).

III. OBSERVATIONS

Observations of the Galaxy were made every $2^{\circ}.5$ in galactic longitude from 15° to 45° (except $22^{\circ}.5$) and every 5° from 45° to 60° . At each longitude, observations were spaced $0^{\circ}.25$ (about two beamwidths) in latitude. In most cases the survey was extended above and below the plane until two successive positions showed no signs of CO emission.

The previous in-plane surveys have found almost no CO emission at negative radial velocities in the longitude range of this survey. Because our spectrometer was not wide enough to make observations of both positive and negative velocities simultaneously, it was therefore decided that, for now, the previous null results would be accepted at negative velocities, and observations were thus confined to positive velocities. For $\ell \geq 40^{\circ}$ neither HI nor CO emission has been observed at velocities with respect to the local standard of rest (v_{LSR}) greater than 100 km sec^{-1} . At these longitudes it was therefore possible to include in the 104 km sec^{-1} bandpass of the spectrometer the entire range of velocities at which emission was expected. Inside $\ell = 40^{\circ}$ the receiver was set to include the highest velocity at which significant emission was found by Burton et al. (1975).

At these latitudes the low velocity (generally $V_{\text{LSR}} < 10 \text{ km sec}^{-1}$) emission, presumably originating in local CO, lay outside the range of the spectrometer, and was consequently not observed. After the completion of this survey, Gordon and Burton (1976) published a high sensitivity in-plane survey that showed some emission at velocities higher than those covered in the present survey at $\ell = 15^\circ$, $17^\circ.5$, and 20° . The omission of this high velocity emission will only affect the conclusions concerning the inner 4 kpc of the Galaxy.

All the spectra were obtained by position switching, i.e., by observing during alternate 30 sec periods at the source position and at a comparison position. The comparison positions ("offs") were checked with considerable care to assure that they were free of any CO emission stronger than 0.3 K (less than half the 3σ noise level of the survey itself). This was done initially by the frequency switching technique in which a comparison spectrum is generated by shifting the frequency of the receiver by half the bandwidth of the spectrometer instead of by moving the telescope to an emission free position. While this scheme usually produces baselines that are inferior to those obtained by position switching, it is ideal for finding emission free regions because it does not involve a second, possibly "contaminated", off position. The comparison positions were further examined

by position switching against each other. The final comparison positions, listed in Table III-1, were typically 5° above and below the galactic plane and 5° apart in longitude.

The receiver was calibrated against a room temperature blackbody by a chopper wheel technique similar to that described by Davis and Vanden Bout (1973). Their method, which corrects for the beam efficiency and for the atmospheric attenuation in a single layer atmosphere, was refined by using a two layer model: an upper layer of O_2 and a lower layer of H_2O . The optical depth of the O_2 was calculated from a standard atmosphere to be 0.21 at the zenith, while the optical depth of water was derived from antenna tippings made at least once every six hours. (See Chin, 1977, for details of the calibration technique.) Observations were generally made on cold, dry winter days when the optical depth of water was between 0.15 and 0.25 at the zenith.

For a given source this calibration procedure gave corrected antenna temperatures, T^* , that were independent of elevation down to the lowest angles above the horizon at which observations were practical (about 20°). Figure III-1 shows a series of measurements, made at different altitudes, of the calibrated intensity from the position of the most

TABLE III-1

Emission-free comparison positions

l ($^{\circ}$)	b ($^{\circ}$)
15	-5
15	+5
20	+5
25	-5
30	-5
35	-5
35	+5
40	-5
40	+5
43	-2
45	+2
45	+3
50	-3
50	+2
55	-5
55	+5
60	-5
60	+5

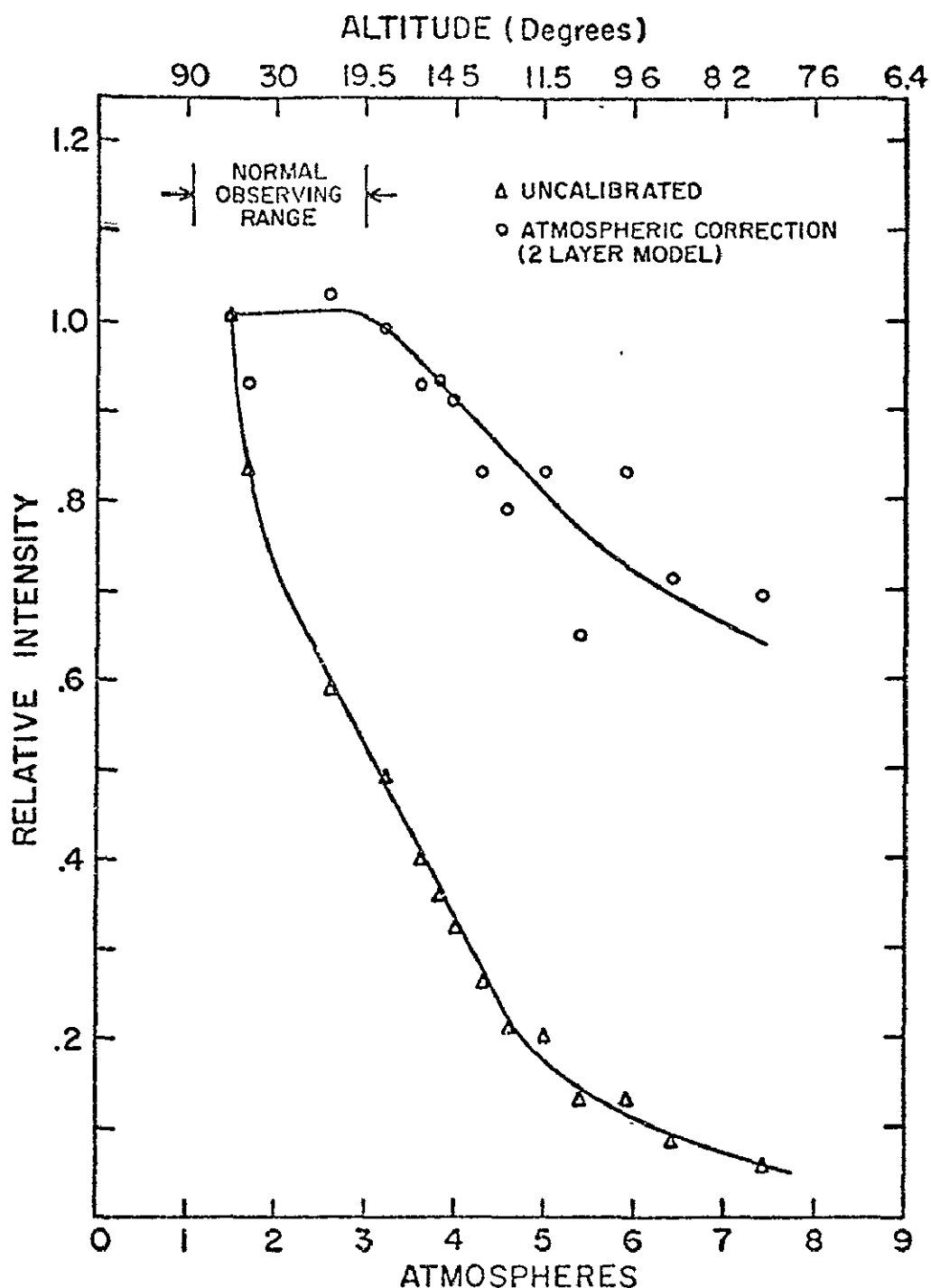


Figure III-1. Measured intensity of CO emission from Orion A relative to value at transit vs altitude of Orion A above the horizon. Both uncalibrated and calibrated data are shown. The calibration is seen to work well over the range of altitudes at which observations are normally made.

intense emission in Orion A, integrated over the line and normalized to the value measured at transit. It can be seen that over the altitudes at which observations are normally made, the calibration procedure compensates extremely well for atmospheric attenuation.

The absolute calibration was checked on several astronomical sources against data taken with the NRAO 36-foot telescope at Kitt Peak. The Kitt Peak results were spatially smoothed to the resolution of the 4-foot telescope and the intensities, integrated over linewidth, were compared. Agreement was found to be within 20% - as good as can generally be obtained by current techniques.

Because the atmosphere contributes different amounts of noise at the signal and comparison positions, the spectra were offset from zero by up to 30° K. In most cases the offset was reduced to less than 10° K by averaging two scans whose comparison positions were displaced roughly equal amounts in altitude on either side of the signal position. In all cases it was necessary to subtract from the spectrum a straight line that was least-squares fit to parts of the spectrum free of emission (usually the ends). Tests on sky regions free of any CO emission demonstrated that this method removed all baseline effects to well below the noise level of this survey. (See, for example, figure III-3, $\ell = 40^\circ$, $b = 3^\circ.25$).

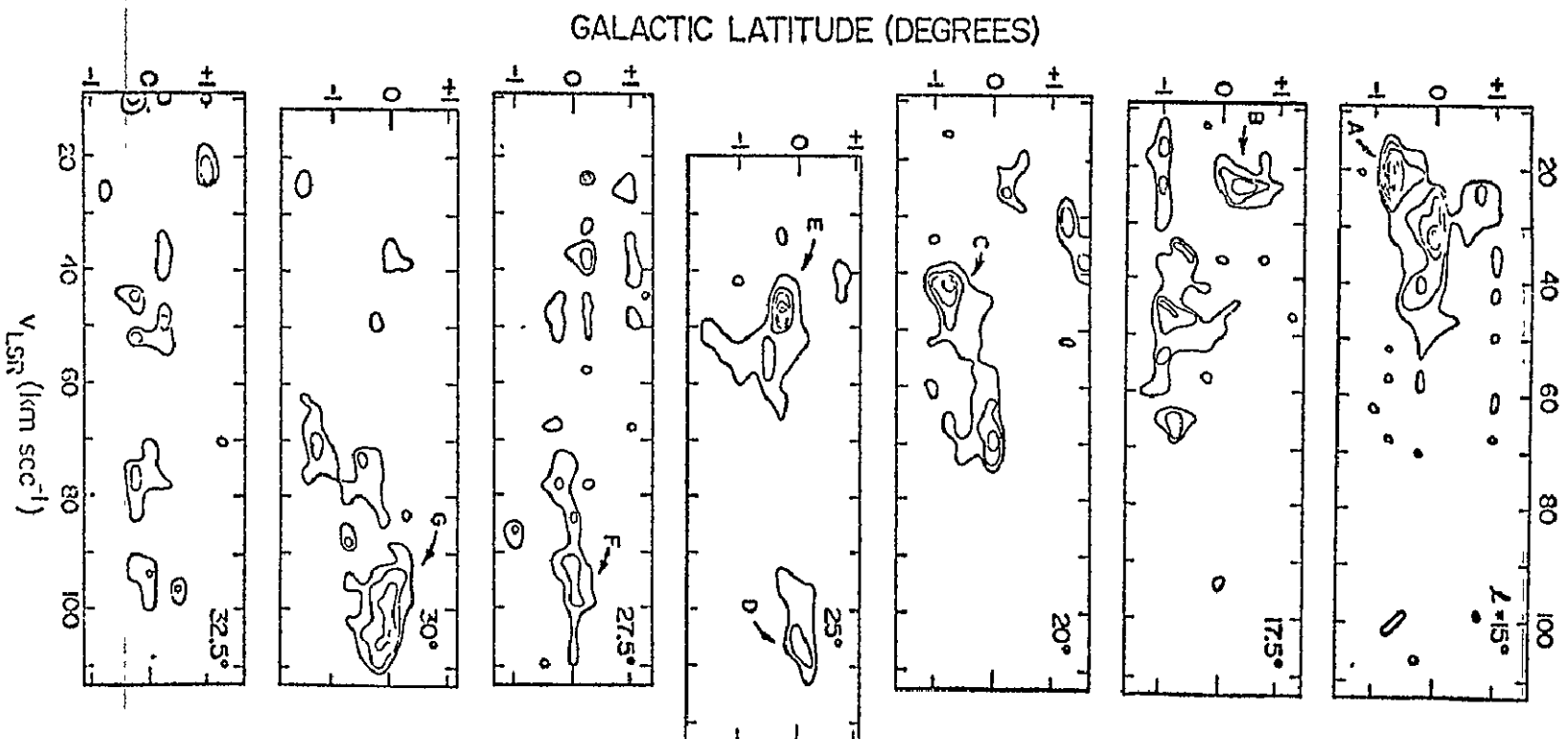
Figure III-2, a summary of the observational data, shows the corrected CO antenna temperature, T^* , at various galactic longitudes as a function of galactic latitude and radial velocity in the local standard of rest. This figure represents a total of 179 spectra. Each spectrum is the result of between 20 and 40 minutes of integration time. The integration time was adjusted, depending on atmospheric conditions, to give a 3σ noise level of about 0.7 K. A sample of individual spectra are shown in Figure III-3.

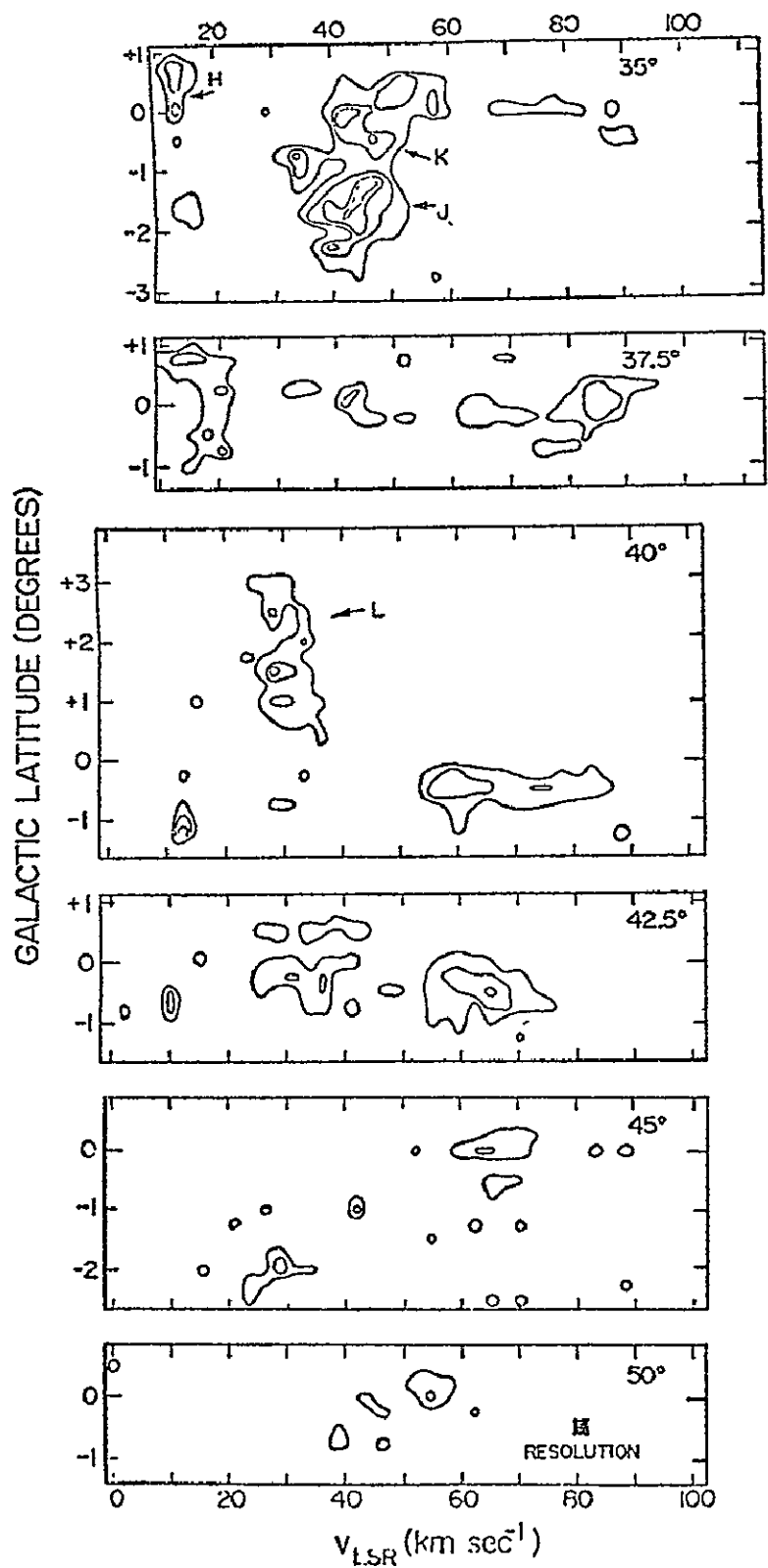
Because we have not mapped closely in latitude, it has not generally been possible to demonstrate clear relationships between the CO clouds shown in Figure III-2 and known astronomical objects. However, we have made preliminary identifications of a few of the more prominent features, indicated by letters on the contour maps and described below.

$l = 15^\circ$: Feature A is M17. This HII region has a large associated molecular cloud that has been thoroughly mapped in CO by Lada (1976).

$l = 17^\circ.5$: Feature B is probably related to M16, a large HII region at $Gl7.0 +0.8$. Georgelin and Georgelin (1976) give the H109 α velocity as 24.5 km sec^{-1} and the H α (optical) velocity as 28.4 km sec^{-1} .

Figure III-2. Distribution of CO emission in the galactic disk. The contour interval is 1 K antenna temperature (corrected for beam efficiency and atmosphere). The area of each plot shows the region actually surveyed in both velocity and latitude. The angular resolution indicated is that of the survey (2 beamwidths). Two additional longitudes were surveyed but showed no emission at the first contour level: $\ell = 55^\circ$ from $b = -0^\circ.75$ to $b = +0^\circ.75$ and $v_{\text{LSR}} = 0$ to 104 km sec^{-1} ; and $\ell = 60^\circ$ from $b = -0^\circ.5$ to $+0^\circ.5$ and $v_{\text{LSR}} = 0$ to 104 km sec^{-1} . The letters indicate features that are described in the text.





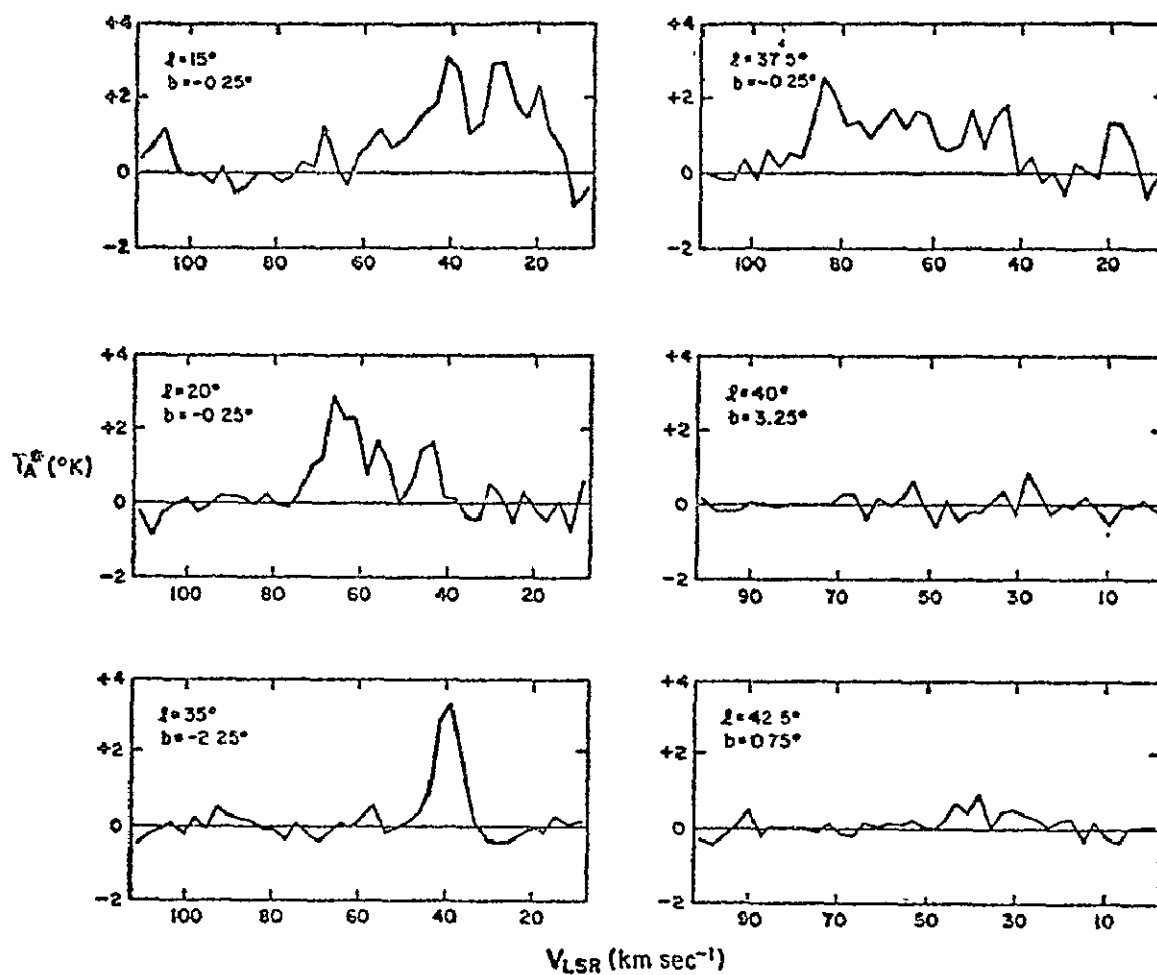


Figure III-3. Some typical spectra.

$\ell = 20^\circ$: Feature C is the HII region G19.7-0.2 which has an H109 α velocity of 43.4 km sec⁻¹ (Georgelin and Georgelin, 1976).

$\ell = 25^\circ$: Reifenstein et. al. (1970) note that this is a complex region. They list two H109 α sources near feature D, namely G24.6-0.2 at 109 km sec⁻¹, and G24.8+0.1 at 114 km sec⁻¹. They give a velocity of 60 km sec⁻¹ for W42 at G25.4-0.2; this source is probably related to feature E. Clark and Caswell (1976) show a SNR at G24.7-0.6 with a diameter of 15 arc minutes, large enough to suggest that it may be related to an object on this map.

$\ell = 27^\circ.5$: Georgelin and Georgelin (1976) give an H109 α velocity of 98 km sec⁻¹ for the HII region at G27.3-0.2, probably feature F.

$\ell = 30^\circ$: Feature G is related to W43, a complex region containing many sources. The CO is most likely associated with the HII region at G29.9-0.0 at an H109 α velocity of 96 km sec⁻¹ (Reifenstein et. al., 1970). There is also a SNR at G29.7-0.2 (Clark and Caswell, 1976).

$\ell = 35^\circ$: There are numerous sources in this region. Feature H is a local dark cloud that has been mapped in H₂CO and OH by Myers (1975). Feature J is the complex W48 region which is related to a large HII region. Feature K is near the SNR W44 located at G34.6-0.5.

$l = 40^\circ$: Feature L is near W45 at G40.5+2.5 and is
probably the HFI region S76 (Sharpless, 1959). S75 is
at G40.2+1.5, and it is possible that a related molecular
cloud appears on this map.

IV. DISTRIBUTION OF CO EMISSION

Before proceeding with a close quantitative analysis, we will attempt to determine the major features of the galactic distribution of CO directly from a careful examination of the contour maps (Fig. III-2). First, we see that there is very little emission for $\ell > 45^\circ$. Since observations in these directions only sample points more than 7 kpc from the galactic center, we can conclude that for $R > 7$ kpc the amount of CO is very small.

Next, in the inner part of the Galaxy we find a discrepancy between the maximum velocities of the HI and CO emissions. For $15^\circ < \ell < 25^\circ$ the maximum HI velocity increases with decreasing longitude, whereas the maximum velocity at which significant CO emission is observed decreases; the high velocity CO is extremely weak or missing. Because of the differential galactic rotation, the highest velocity emission at a given longitude originates closest to the galactic center. Thus the missing emission for $\ell < 25^\circ$ implies a lack of CO inside of the $\ell = 25^\circ$ tangent point, i.e. for $R < 4$ kpc. So, we can see that the CO emission is concentrated in a molecular ring between about 4 and 7 kpc from the galactic center.

To estimate the out-of-plane distribution of CO emission, all the spectra at the same latitude were integrated over frequency and averaged together. The result (see Figure IV-1) shows that the CO plane is $1^{\circ}.2$ thick (FWHM) and is displaced $0^{\circ}.3$ towards negative latitude. Taking $R = 6$ kpc as the average radius of the molecular ring and ignoring, for now, radiation from the far side of the Galaxy, we find that the distance from the sun to the ring is between 4 kpc (toward the galactic center) and 8 kpc (toward the tangent point). Using 6 kpc as typical of the distance to the ring, the angular thickness of the molecular disk corresponds to about 130 pc (FWHM) and the displacement to 30 pc.

In order to analyze the survey data more quantitatively we will assume that although local observations indicate that CO is generally found in clouds in which it is optically thick, the Galaxy is optically thin in the sense that the clouds have a sufficiently sparse distribution that radiation from distant clouds is not obscured. While there are usually several clouds along any given line of sight passing through the molecular ring, differential galactic rotation will generally Doppler shift the radiation from distant clouds to frequencies at which more nearby matter is completely transparent. The best evidence that the

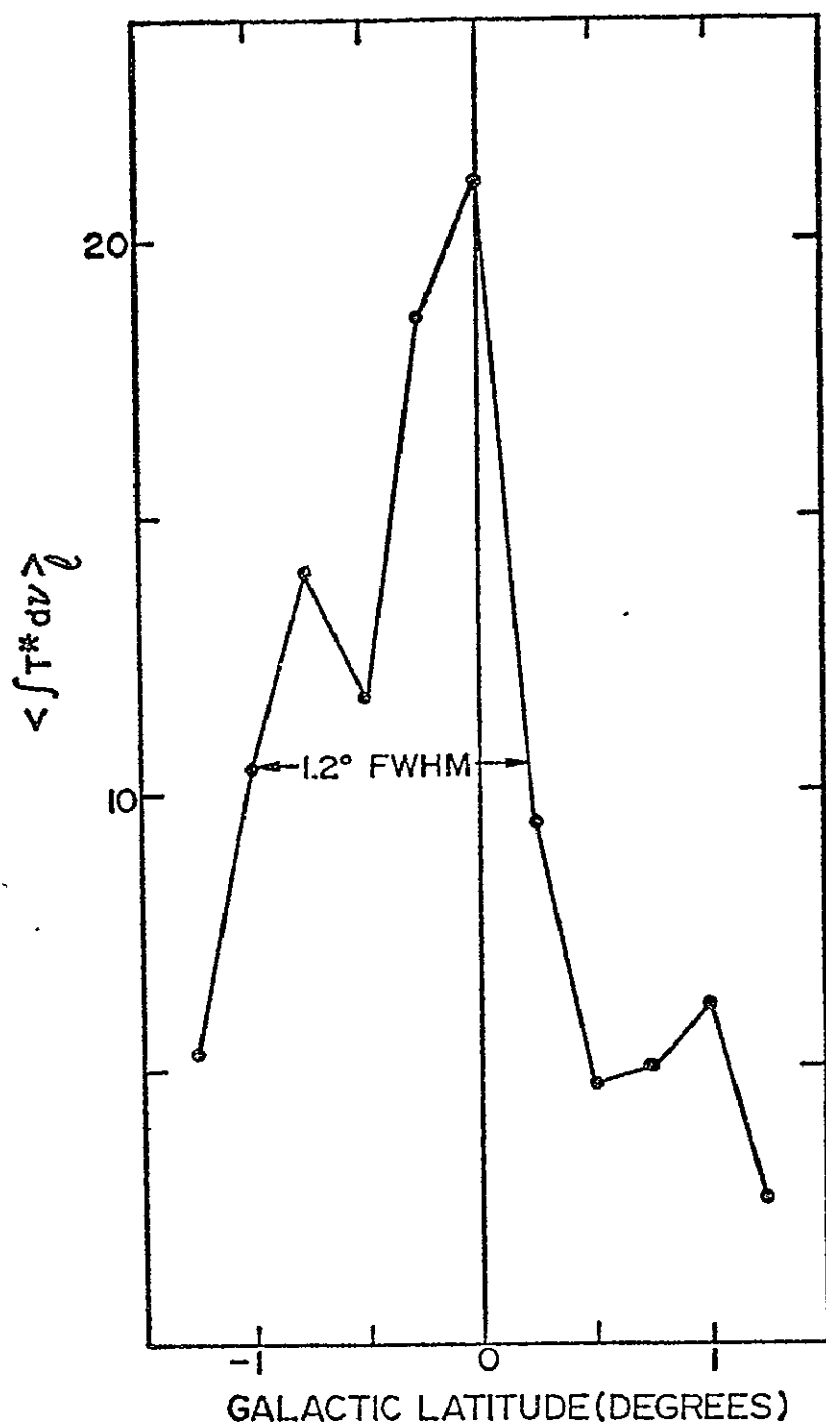


Figure IV-1. Antenna temperature integrated over frequency and averaged over galactic longitude (in °K MHz) vs galactic latitude.

molecular clouds are in fact sparse enough not to obscure each other is the appearance of the high resolution spectra taken with the NRAO 36-foot telescope as a series of well defined features separated by velocities with no emission (Burton et al., 1975). If there were many clouds along the line of sight with similar Doppler velocities, the emission would not be likely to fall to zero between clouds.

To convert the Doppler velocities of the CO clouds into galactic positions, a simplified model of galactic rotation has been used. Specifically, it is assumed that the large scale structure of the Galaxy is azimuthally symmetric and that the only motion is circular rotation. Although the azimuthal symmetry of the model will suppress evidence for spiral arms, density waves, etc., it will have only a small effect on the radial distribution derived here. The fluctuations in the observed rotation curve, sometimes attributed to non-circular "streaming" motions, are generally less than 10 km sec^{-1} . If the simplified circular rotation model is used, these non-circular motions can produce as much as a 2.5 kpc error along the line of sight; however, the corresponding error in galactocentric distance is never more than 1.3 kpc over the region of sky covered in the present survey. For $4 < R < 8 \text{ kpc}$, the region where

almost all the CO emission is concentrated, the error is less than 0.7 kpc.

Because of its analytic convenience, the rotation curve from Burton (1971) has been adopted. This curve, derived from 21-cm data, gives the velocity as a function of galactic radius to be

$$V(R) = 250.0 + 4.05 (R_0 - R) - 1.62 (R_0 - R)^2 \quad (\text{IV-1})$$

where $V(R)$ is in km sec^{-1} , R is the galactic radius in kpc, and $R_0 = 10$ kpc is the distance of the sun from the galactic center. For $3 < R < 10$, this curve differs from the Schmidt curve by less than 3 km sec^{-1} and from the curve derived by Gordon and Burton (1976) from CO data by less than 2 km sec^{-1} . These discrepancies are less than the observational uncertainties in the rotation curve due to small scale fluctuations and the criterion used to determine the maximum Doppler velocity at a given longitude.

In the region interior to the solar circle covered by this survey, it is impossible, even with simplifying assumptions about galactic rotation, to assign a unique position in the Galaxy to each observed feature; along a given line of sight the two points with the same distance from the galactic center have the same velocity component towards the sun. The galactocentric distance of a cloud is thus uniquely determined, but the distance from the sun is ambiguous.

Since it is impossible to establish the locations of individual clouds, we have instead fit the data to simple stochastic models. The CO is assumed to be contained in many small, randomly distributed clouds. We take the average number of clouds in the volume element dV , radiating in the $J = 1 \rightarrow 0$ line of CO with an intensity between B and $B + dB$ in the frequency interval between ν and $\nu + d\nu$, measured in the sun's local standard of rest, to be given by a distribution function

$$g(R, z, B, \nu) dV dB d\nu \quad (\text{IV-2})$$

where R is galactic radius and z is height above the galactic plane. If the clouds are all of the same radius, a , and if g does not vary significantly over a distance comparable to a , then the average number of clouds intersected by a given line of sight between a distance s and $s + ds$ from the sun will be given by

$$f(R, z, B, \nu) ds dB d\nu = \pi a^2 g(R, z, B, \nu) dV dB d\nu \quad (\text{IV-3})$$

where R and z are now functions of s and the galactic coordinates l and b .

If we ignore the distant wings of the line and assume that the CO clouds are optically thick over the entire CO line, then we will see only the nearest cloud along the

line of sight emitting at a given frequency. Therefore, the expectation value of the observed intensity will be

$$\begin{aligned}
 I(\ell, b, \nu) &= \int_0^\infty [\text{probability there is no cloud nearer} \\
 &\quad \text{than } s \text{ to the sun}] \times \left\{ \int_0^\infty [\text{probability} \right. \\
 &\quad \text{of cloud of brightness } B \text{ at a} \\
 &\quad \left. \text{distance } s] B \, dB \right\} \, ds \\
 &= \int_0^\infty [\exp(-\tau_\nu(s))] \times \epsilon_\nu(R, z) \, ds \quad (\text{IV-4})
 \end{aligned}$$

where $\tau_\nu(s) = \int_0^s \int_0^\infty f(R, z, B, \nu) \, dB \, ds$ is the average number of clouds out to a distance s , and $\epsilon_\nu(R, z) = \int_0^\infty B f(R, z, B, \nu) \, dB$. Equation IV-4 is the same as the solution of the equation of transfer in local thermodynamic equilibrium with an effective optical depth τ_ν and an effective volume emissivity ϵ_ν .

If, as we have assumed, there is a low probability for more than one cloud to be along a given line of sight at a given frequency, i.e. $\tau \ll 1$, then we can ignore the exponential term in Equation IV-4 and treat the Galaxy as effectively optically thin. Then we have

$$I(\ell, b, \nu) = \int_0^\infty \epsilon_\nu(R, z) \, ds \quad . \quad (\text{IV-5})$$

Finally, we define the effective integrated volume emissivity as

$$\epsilon(R, z) = \int \epsilon_v(R, z) dv , \quad (\text{IV-6})$$

where the integral extends over the CO line. It is this last function that we attempt to determine.

Because it is usual in radio astronomy to express intensity in temperature units as $T^* = (c^2 / 2kv^2) I$, we will use compatible units for the effective emissivity and define

$$\epsilon^* = (c^2 / 2kv^2) \epsilon \quad (^\circ\text{K Mhz kpc}^{-1}) . \quad (\text{IV-7})$$

The effective emissivity will always be expressed in these units in this thesis and we will generally drop the asterisk.

For our first stochastic model of the Galaxy, the effective integrated volume emissivity was assumed to be a gaussian in z , specifically,

$$\epsilon(z, R) = \epsilon_0 \exp \left[\frac{-(z-z_0)^2 (\ln 2)}{z_{1/2}^2} \right] \quad (\text{IV-8})$$

where z is the distance above the galactic plane, R is the distance from the galactic center, and the displacement z_0 , the half-thickness at half power $z_{1/2}$, and the central emissivity ϵ_0 , are all functions of R that characterize the CO distribution.

To obtain the most likely values of the parameters $\epsilon_0(R)$, $z_{1/2}(R)$, and $z_0(R)$ the Galaxy was divided into concentric rings of width 0.5 kpc. Initial estimates of the three parameters were chosen for each ring. Then, using the model emissivity function (Eq. IV-8) and the assumption of optical thinness, model spectra were computed for each latitude and longitude that was actually observed. The model spectra were compared with the observed spectra and the parameters were varied in each ring to obtain the best least-squares match of the model and observed spectra. Details of the fitting procedure and the computation of model spectra are given in Appendices C and D respectively. In rings with little or no CO there was insufficient data to fit all the parameters accurately. In that case the CO emission was assumed to be symmetric about the galactic plane, i.e., z_0 was constrained to zero. The resulting parameters as functions of R are shown in Figure IV-2 (left).

Proper convergence of the fit was assured in several ways. First, the fitting, which was done by an iterative method, was continued until the change in each of the parameters from one iteration to the next was less than 2% of the calculated error in the parameter. Second, the entire fitting was repeated several times using different initial estimates for the parameters, including $\epsilon_0 = 0.2$ and 1;

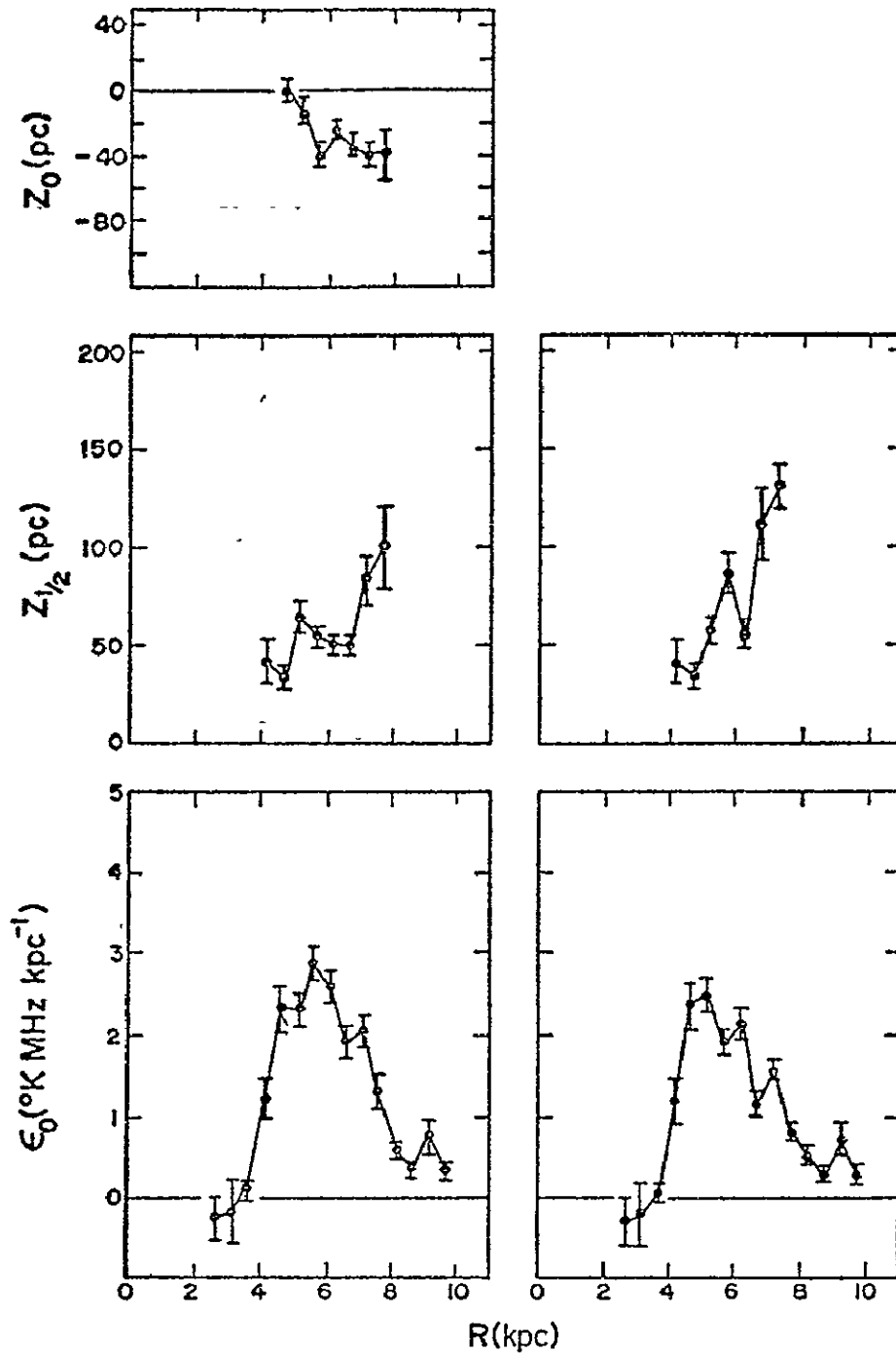


Figure IV-2. Results of fitting data to a gaussian in z . Left: Displacement, half-thickness, and central emissivity of the CO disk as functions of galactocentric distance. Right: Half-thickness and central emissivity with the displacement constrained to zero.

$z_{\frac{1}{2}} = 10 \text{ pc}, 50 \text{ pc}, 200 \text{ pc}, \text{ and } 1000 \text{ pc};$ and $z_0 = 0 \text{ pc}$ and 1000 pc . In every case in which the fit converged, it converged to the same value.

Finally, as an overall check on the algorithm, the fitting was tested against a simulated version of the Galaxy, made up of randomly distributed clouds. The probability density for an individual cloud to fall at the galactocentric cylindrical coordinates (R, θ, z) was given by

$$\frac{1}{(2 \pi \sigma_R \sigma_z)} \exp\left(-\frac{(R-R_0)^2}{2\sigma_R^2}\right) \times \exp\left(-\frac{(z-z_0)^2}{2\sigma_z^2}\right) \times U(\theta) \quad , \quad (\text{IV-9})$$

where $U(\theta)$ is a normalized uniform density between 0° and 360° , $R_0 = 6 \text{ kpc}$, $\sigma_R = 40 \text{ pc}$, and $\sigma_z = 60 \text{ pc}$ (corresponding to $z_{\frac{1}{2}} = 85 \text{ pc}$). The clouds were all taken to have the same temperature, size, and linewidth. The size and linewidth were taken to be sufficiently small compared to the resolution of the telescope that the clouds can be treated as essentially pointlike with delta function emission lines.

From the simulated galaxy, spectra were constructed corresponding to what would be observed with the velocity and angular resolutions of the 4-foot telescope. These

spectra, which were saved by the computer in exactly the same format as the actual spectra, were then input to the least-squares fitting program as if they were real spectra. Figure IV-3 shows the results of the fitting for a simulated galaxy. In this case there were typically 15 clouds per resolution element in the portions of the spectra that correspond to a galactic position near the center of the simulated molecular ring. (Models with a much smaller number of clouds are considered in Appendix C.) It can be seen that the fitted values match the input distribution quite well and that the calculated error bars are a good indication of the actual errors.

The actual data were also fit to a second, more restricted, model in which z_0 was constrained to zero everywhere. For $R < 5.5$ kpc the resulting $\epsilon_0(R)$ and $z_0(R)$, shown in Figure IV-2 (right), do not differ significantly from the results of the previous fit. However, at greater radii, where the galactic plane is depressed according to Figure IV-2 (left), there is a slight increase in $z_{1/2}$. This is expected since the CO is typically further from $z = 0$ than from the true z_0 .

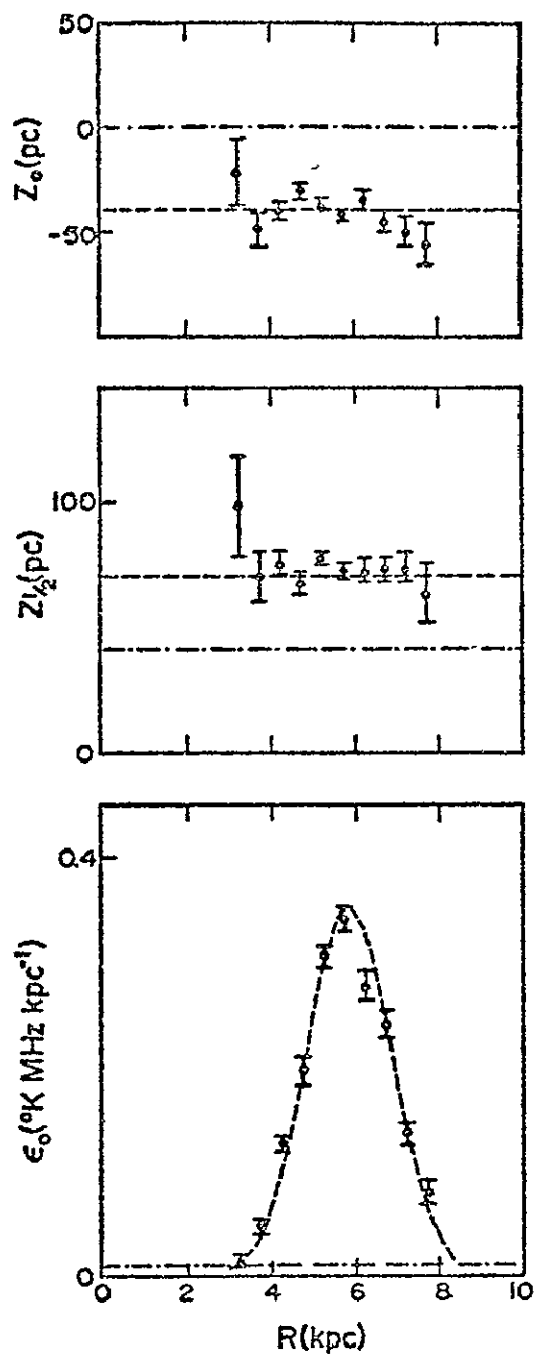


Figure IV-3. Results of least-squares fitting to simulated galaxy. The dashed lines (---) indicate the underlying distribution used in simulating the galaxy. The dash-dot lines (-.-.-) show the original parameter estimates used in the fitting.

Several checks were made to verify that the offset in z_0 shown in Figure IV-2 was not the result of a single large object far from the plane. The fitting was repeated twice, in one case ignoring the data for $\ell = 35^\circ$, the latitude with the largest concentration of emission below the plane, and in the other case the data for $\ell \leq 25^\circ$, where the depression in the plane is most apparent to the eye in the contour plots (Fig. III-2). In neither case did the offsets (see Figure IV-4) differ significantly from those obtained when all the data were included.

Finally, the model emissivity function was changed to an exponential:

$$\epsilon(R, z) = \epsilon_0 \exp \left[- \frac{|z - z_0| (\ln 2)}{z_{1/2}} \right] \quad (\text{IV-10})$$

where ϵ_0 , z_0 , and $z_{1/2}$ are again functions of R . The best values of the parameters were determined as before with the results as shown in Figure IV-5.

If we compare these results with those from the gaussian model, we find that there is very little change in the shape of the curves. We will now make a more quantitative com-

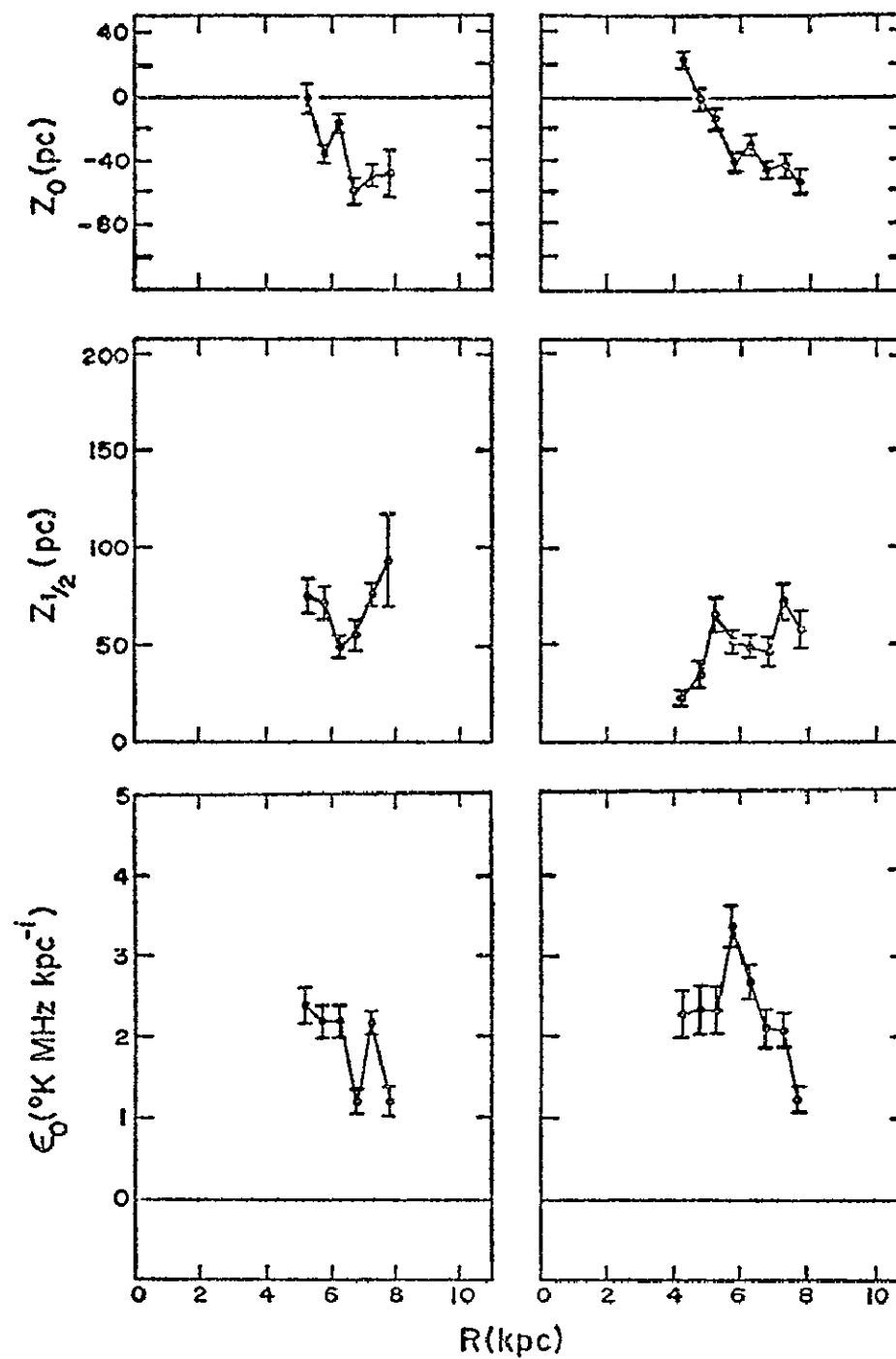


Figure IV-4. Left: Results of fitting only the data for $\ell \geq 27^{\circ}.5$. Right: Results of fitting data for $\ell \neq 35^{\circ}$.

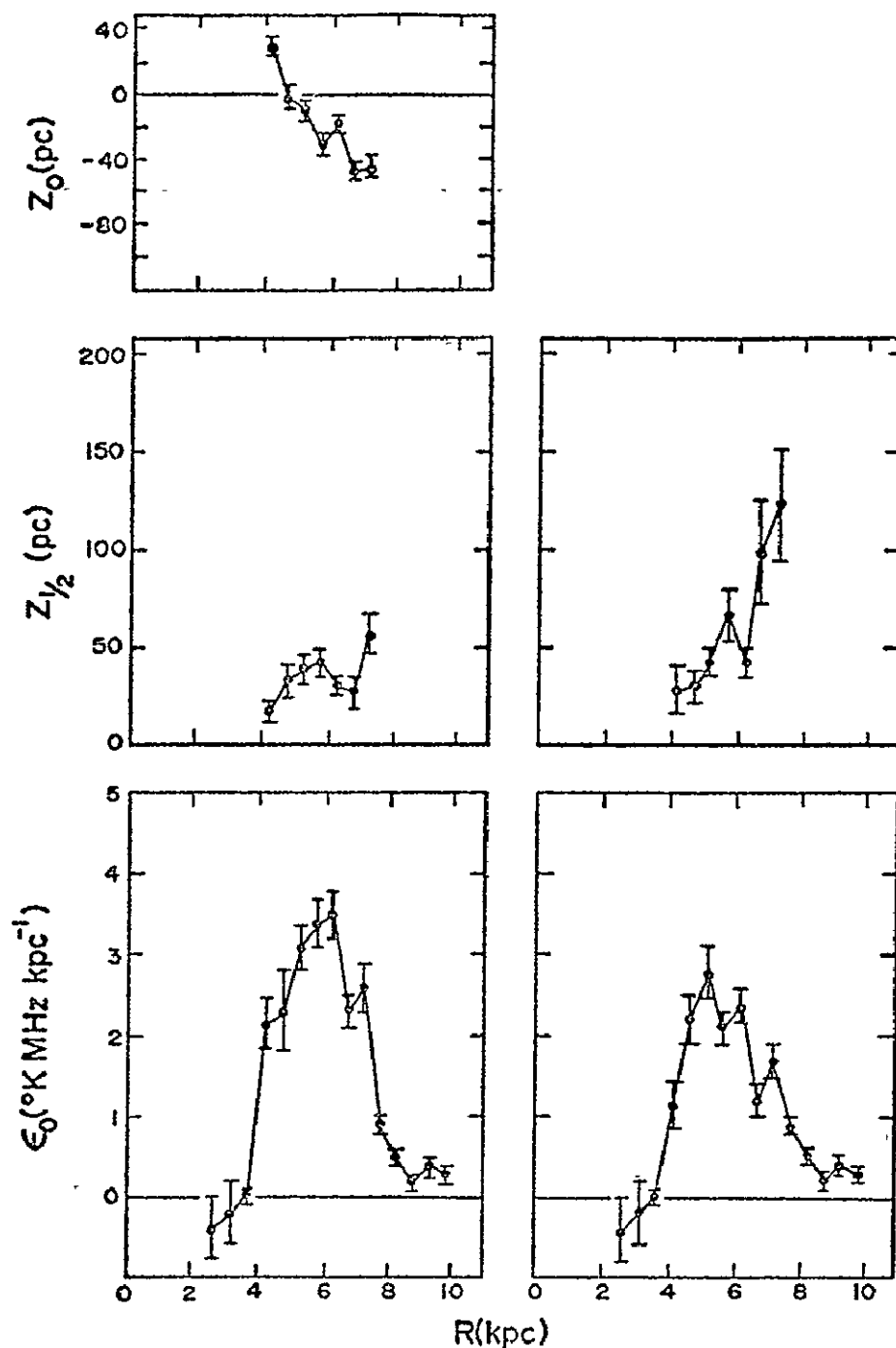


Figure IV-5. Results of fitting the data to an exponential in z . Left: Displacement, half-thickness, and central emissivity of CO disk as functions of galactocentric distance. Right: Half-thickness and central emissivity with the displacement constrained to zero.

parison of the models with three parameters (Figs. IV-2 left, and IV-5, left) in the region $4.5 < R < 7.5$, where all the parameters can be determined in both the gaussian and exponential models. There is no significant change in the offset between the models: for the gaussian model, the average offset is $\langle z_{o,g} \rangle = -24.3$ pc, while for the exponential model we get $\langle z_{o,e} \rangle = -25.5$ pc. There is however a significant change in the $z_{\frac{1}{2}}$ curve. Computing the average value of the ratio of $z_{\frac{1}{2}}$ for the gaussian model to that for the exponential model, we obtain $\langle z_{\frac{1}{2},g} / z_{\frac{1}{2},e} \rangle = 1.45$. Similarly, for the central emissivity we get $\langle \epsilon_{o,g} / \epsilon_{o,e} \rangle = 0.84$. If we define the surface brightness of the Galaxy as

$$\sigma(R) = \int_{-\infty}^{\infty} \epsilon(R, z) dz \quad , \quad (\text{IV-11})$$

then for the gaussian model

$$\sigma_g = (\pi / \ln 2)^{\frac{1}{2}} z_{\frac{1}{2}} \epsilon_o \quad , \quad (\text{IV-12})$$

and for the exponential model

$$\sigma_e = (2 / \ln 2) z_{\frac{1}{2}} \epsilon_o \quad . \quad (\text{IV-13})$$

The value of σ_g / σ_e averaged over radius is $\langle \sigma_e / \sigma_g \rangle = 0.88$. Thus the change in $z_{\frac{1}{2}}$ between the two models is just about consistent with identical surface brightness.

The error bars in Figures IV-2, -3, -4, and -5 indicate the range over which the parameters may be varied without significantly affecting the quality of the fit as measured by σ^2 , the mean square deviation of the model spectra from the observed spectra. (See Appendix C for details.) If the model matched the data very well, and σ^2 were thus small, then a small change in a parameter would result in a significant fractional change in σ^2 and consequently the error bars would be small. Thus the error bars measure, in effect, the inability of the model spectra to exactly reproduce the observations. Deviations of the observed spectra from the model spectra result primarily from random fluctuations in the distribution of molecular clouds. In principle, noise and inadequate baseline removal could also contribute to the deviations, but actually, they are insignificant for $R > 3.5$ kpc.

Summarizing this section, by fitting the survey data with simple models of the distribution of CO emission, we have determined the thickness, the offset from $z=0$, and the central emissivity of the CO disk of the Galaxy. The results are insensitive to the detailed form of the model emissivity function. They are also not strongly influenced by the exclusion of a small part of the data. Lastly, we

note that the fitting has confirmed the qualitative estimates made at the beginning of this section concerning the existence of a molecular ring between about 4 and 7 kpc from the galactic center, and the thickness and offset of the CO layer.

In view of the similarity of the results obtained by using either an exponential or a gaussian function for the model z-distribution of CO emission, our analysis alone presents no compelling reason to adopt one form over the other. However, there are arguments favoring the gaussian distribution: first, it is predicted from simple hydrodynamic theory (Oort, 1965) and second, it seems to be a good representation of the distribution of HI clouds (Schmidt, 1957). We have chosen therefore to use the results of the gaussian model (Figure IV-2, left) in the succeeding discussion.

V. DISCUSSION

The major results of the analysis of Chapter IV are the thickness of the galactic CO disk, its offset from the conventional galactic plane, and its central emissivity -- all as functions of galactic radius. In this chapter we will discuss these results and, in particular, their relationship to previous observations of the Population I component of the Galaxy.

Central emissivity

The central emissivity, $\epsilon_0(R)$, (see Fig. V-1 which repeats the results of Fig. IV-2) is strongly peaked in a molecular ring lying between 5 and 7 kpc from the galactic center. Although a similar result had been obtained by the in-plane surveys, our conclusion is considerably stronger in that it is clear that we have properly located the position of the CO emission with respect to $b = 0^\circ$.

When our survey is compared in detail with the in-plane surveys, we find that if the emissivities measured by Gordon and Burton (1976) and Scoville and Solomon (1975) are normalized to our results in the molecular ring, then the $\epsilon_0(R)$ curves agree reasonably well for $R > 4$ kpc (see Fig. V-1). However, inside this radius, where the

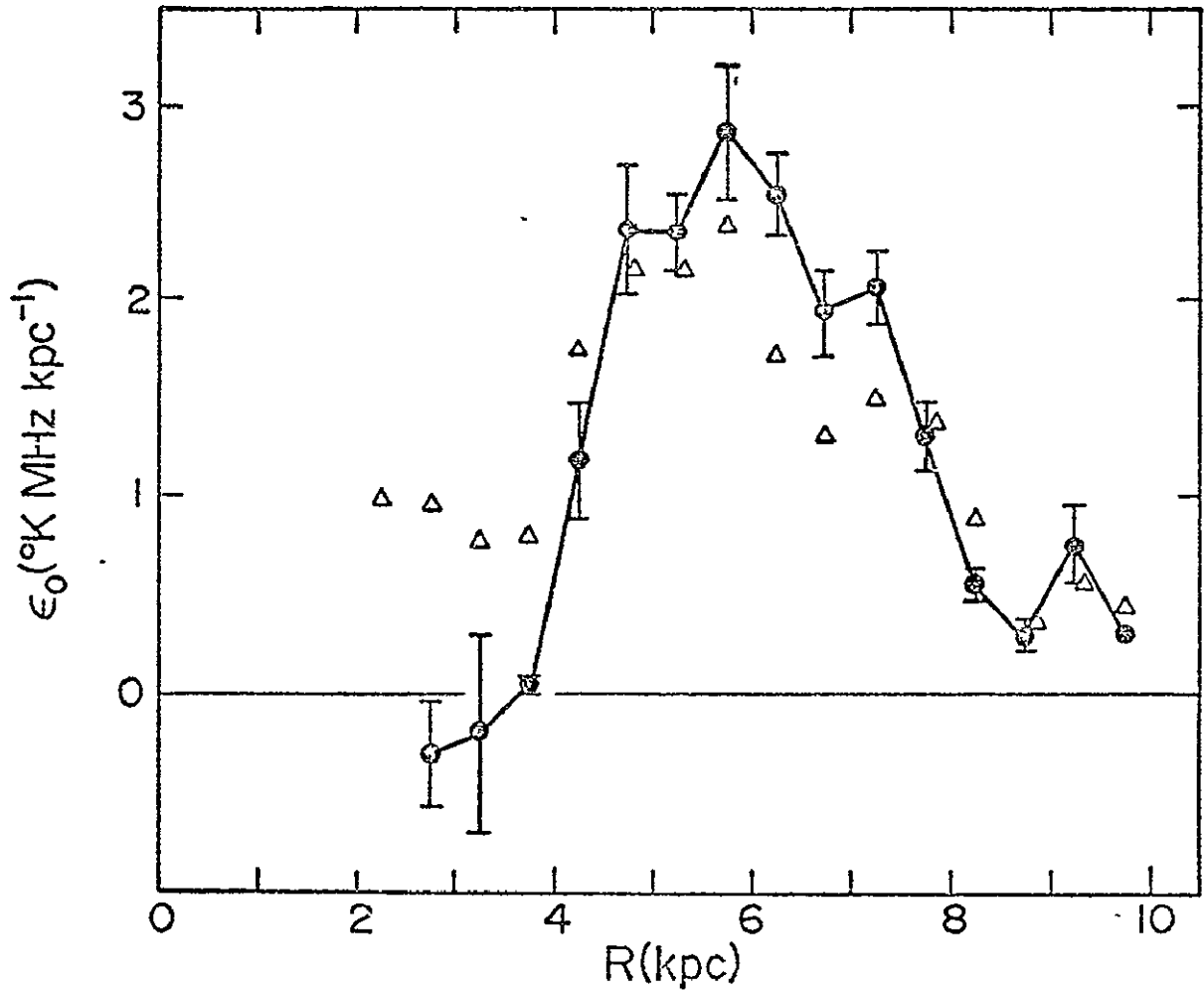


Figure V-1. Emissivity of CO vs galactic radius. Filled circles: present work. Open triangles: Gordon and Burton (1976) scaled down by a factor of 4.3 to agree with the present work in the molecular ring (see discussion in text) .

emission is weak, there is considerable disagreement: the Gordon and Burton results are about double those of Scoville and Solomon, while our survey shows no emission. Although Scoville and Solomon used the Schmidt rotation curve; Gordon and Burton used a curve derived from CO data; and we have used the curve given in Eq. IV-1, we have shown in Chapter IV that these differences in the choice of rotation curve are unlikely to cause large errors in the derived radial distributions. As was noted in Chapter III, Gordon and Burton found several clouds at $l \leq 20^\circ$ at velocities higher than those covered in our survey. It is the emission from these clouds that appears to be the source of the discrepancy.

While it is, of course, desirable to have as wide a velocity coverage as possible and the Gordon and Burton survey is thus somewhat more complete than ours for $R < 4$ kpc, one can not conclude that the Gordon and Burton results for ϵ are necessarily a more accurate representation of the actual CO distribution. If, as was assumed by all three surveys (Gordon and Burton, Scoville and Solomon, and this thesis), the galactic rotation were purely circular, and the Galaxy were circularly symmetric, then similar results would be obtained using data from any velocities and longitudes covering a given range of galactocentric distances.

The fact that the surveys obtained different results by observing at velocities and longitudes that were different but were predicted by the rotation curve to correspond to the same galactic radius suggests that either the rotation curve does not actually predict the correct radius or that the Galaxy is not circularly symmetric. In fact, we know from 21 cm data (Burton, 1974) that for $R < 4$ kpc there is a serious breakdown in the circular rotation hypothesis and probably also in circular symmetry. We therefore conclude that the simplified models used by all observers for analyzing CO emission data are simply inadequate for $R < 4$ kpc and that all the results in that region must be considered highly speculative.

For $R > 4$ kpc, the situation is quite different. There, as was demonstrated in Chapter IV, the elimination of the data from a wide range of longitudes has only a nominal influence on the derived distribution. In addition, all the CO surveys agree reasonably well in that region in spite of their different samplings in longitude. The 21 cm data also indicate that our model is reasonable for $R > 4$ kpc. As was discussed in Chapter IV, the small scale fluctuations in the observed 21 cm rotation curve of about 10 km sec^{-1} , presumably caused by non-circular motions, are not important for our purposes. The well-

known deviation between the northern and southern hemisphere rotation curves, also caused by non-circular motions, is again about 10 km sec^{-1} and, for present purposes, indicates only a minor deviation from circular rotation.

Because very different beamwidths were employed, we can not expect the spectra from this survey, done with a 4-foot telescope, to agree in detail with the spectra from the previous in-plane surveys, all of which were done with the NRAO 36-foot telescope; a 4-foot spectrum should be approximately the average of all the 36-foot spectra taken within the 4-foot's beam. If the 4-foot's resolution is small enough that at a given velocity there is no systematic large-scale variation in the CO distribution over its beam, then a 36-foot spectrum taken at the center of the 4-foot's beam will be typical of all the spectra that cover the beam, and the antenna temperatures recorded at a given velocity and position on the 4-foot and 36-foot telescopes should agree on the average. For $b = 0^\circ$, antenna temperatures from the two telescopes can in fact be compared since the 4-foot beam corresponds to only about 50 pc even at the most distant region surveyed (about 20 kpc away) -- considerably less than the 120 pc (FWHM) width of the CO layer. When integrated over all velocities

to eliminate the effects of different velocity resolutions, we can expect that on the average the 4-foot and 36-foot spectra will agree.

When this comparison is actually made, no two surveys agree: our results fall roughly a factor of 1.4 below those of Scoville and Solomon (1975), a factor of 2 below those of Burton et. al. (1975), and a factor of 3 below those of Gordon and Burton (1976). Since the absolute calibrations of the 4-foot and NRAO telescopes agree reasonably well (see Chapter II), and all the surveys done at NRAO used the same calibration procedure, this discrepancy does not appear to be an effect of calibration.

The most likely explanation is the existence of an apparent weak substratum of CO emission covering much of the positive velocity region permitted by the rotation curve, and contributing most of the integrated intensity in the Gordon and Burton high sensitivity survey, but only partially detected in the other surveys. This emission has an antenna temperature of 1.2 K and, for $\ell \leq 35^\circ$, covers about 80 to 100 km sec⁻¹ of each spectrum, contributing about 100°K km sec⁻¹ to the integrated antenna temperature -- about 60% of the average total integrated antenna temperature. Subtracting this substratum from

Gordon and Burton's results reconciles them with ours to within about 20% .

The reason that the weak emission was not fully detected in all the surveys is probably related to their differing velocity coverages: the present survey covers 104 km sec^{-1} in each spectrum, Scoville and Solomon covered 166 km sec^{-1} , and Burton et. al. and Gordon and Burton covered 333 km sec^{-1} . In both the present survey and in Scoville and Solomon, because the weak emission covered such a large fraction of the spectrum and had an intensity roughly at the detection limit, it was presumably indistinguishable from the instrumental baseline and was subtracted out in the data analysis. Although we have not detected this weak emission, and the present survey is hence biased toward the more intense molecular sources, the close agreement in the radial distribution of all the surveys strongly implies that the clouds we are seeing do provide a reasonably faithful measure of the overall CO distribution.

Thickness of the CO layer

The thickness of the CO layer, which had not been previously measured as a function of galactic radius, is roughly constant in the region of the molecular ring, but increases by a factor of about two from $R = 7$ to 8 kpc . However, in

view of the large error bars associated with the measurement for $R > 7$ kpc, the increase is not yet firmly established. An average $z_{\frac{1}{2}}$, determined by weighting the thickness values for each ring by the area of the ring, yields $\langle z_{\frac{1}{2}} \rangle = 59$ pc.

Data on the average thicknesses of other Population I objects, which have already been presented in the Introduction and are summarized in Table V.1, show that, with the exception of HI and pulsars, the thicknesses of all the Population I objects agree rather closely. Pulsars, which have an anomalously broad distribution, presumably as a result of their large velocities at formation, will not be considered further.

Only for HI, SNR's, and now CO is detailed information on the radial variation of the thickness available over a large fraction of the Galaxy. A comparison of these results (Figure V-2) again shows a general agreement between SNR's and CO, while the HI curve has a similar shape but about double the value. Thus there appears to be a natural division of the Population I component into two classes: 1) objects more or less directly associated with star formation or young stars and 2) HI gas and clouds.

TABLE V.1

Thickness of Population I Disk

Tracer	Thickness ($z_{1/2}$ in pc)	Reference
HI gas	140	Baker and Burton (1975)
HI clouds	95	Baker and Burton (1975)
HII gas	40-80	Lockman (1976); Gordon <u>et al</u> (1972)
HII regions	40	Burton <u>et al</u> (1975)
SNR	67	Ilovaisky and Lequeux (1972)
Pulsars	330	Seiradakis (1977)
CO	59	present work
γ -rays	??	

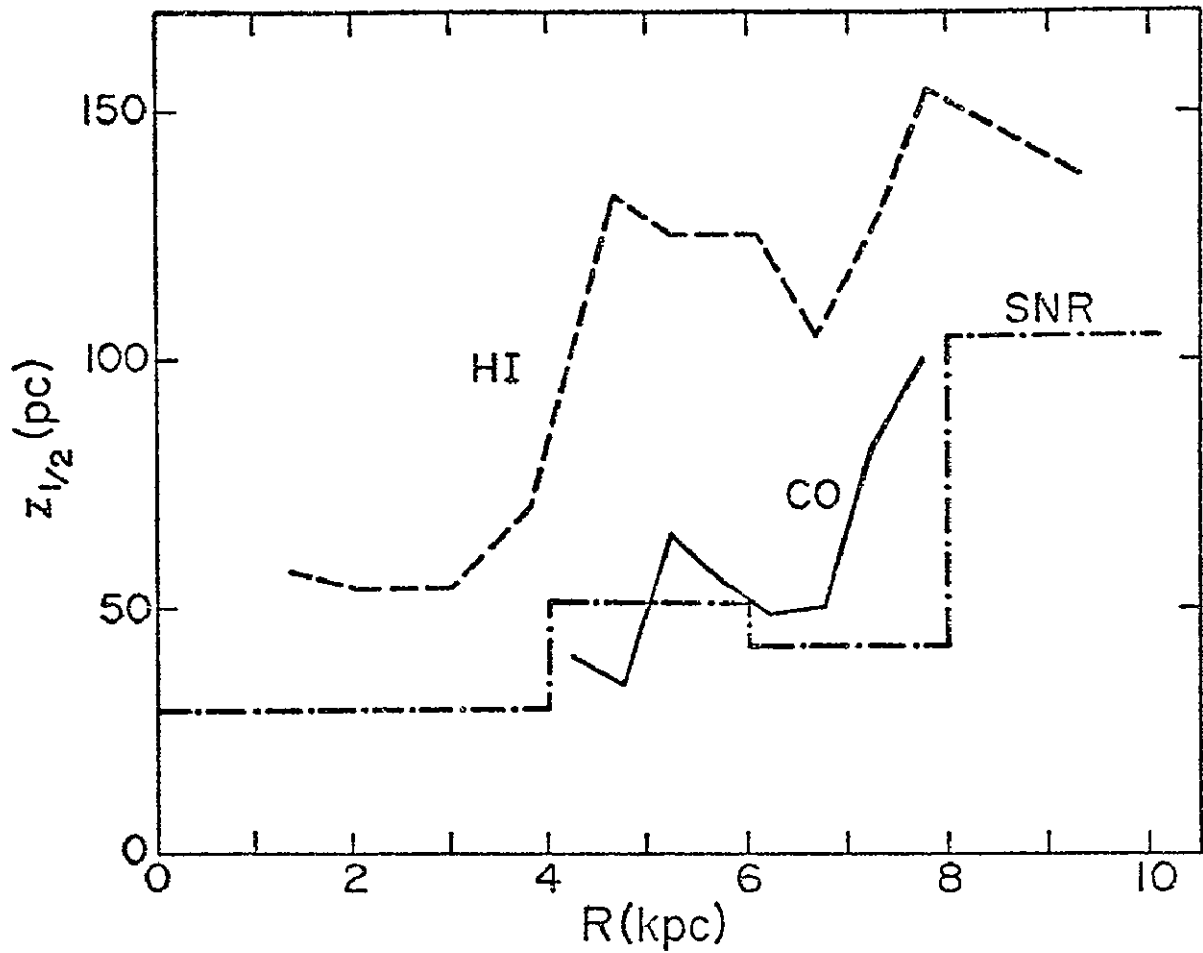


Figure V-2. Half-thickness of the galactic disk for CO, HI, and SNR's. The HI thickness, which includes both clouds and diffuse gas, has been measured at the tangent point by Jackson and Kellman (1974). The $z_{1/2}$ for SNR's has been derived from the value of $\langle z^2 \rangle^{1/2}$ given by Ilovaisky and Lequeux (1972).

As was discussed in the Introduction, a similar correlation of the radial distributions of all Population I objects -- again except for HI -- has already been noted by other authors. Based on the radial distribution and the limited thickness data available to him, Stecker (1976) has proposed that atomic hydrogen be regarded as a new, distinct, Population 0. The detailed thickness data presented here lends additional strong support to this classification scheme.

Combining the central emissivity and thickness data, we can now calculate the CO surface emissivity of the Galaxy as $z_{\frac{1}{2}} \epsilon_0 (\pi / \ln 2)^{\frac{1}{2}}$. For $R > 8$ kpc, where there is no thickness measurement, we have adopted $z_{\frac{1}{2}} = 100$ pc. The result (Fig. V-3) implies that, because of the increasing thickness at large R , the molecular ring is not as sharply peaked as previous surveys had implied, but extends from 5 to 8 kpc.

Offset of the CO disk from the conventional plane

The position of the CO disk, $z_0(R)$, is about 40 pc below the conventional galactic plane in the region of the molecular ring. It was previously noted that the contour plots (Fig. III-2), the gross average of the data over all longitudes

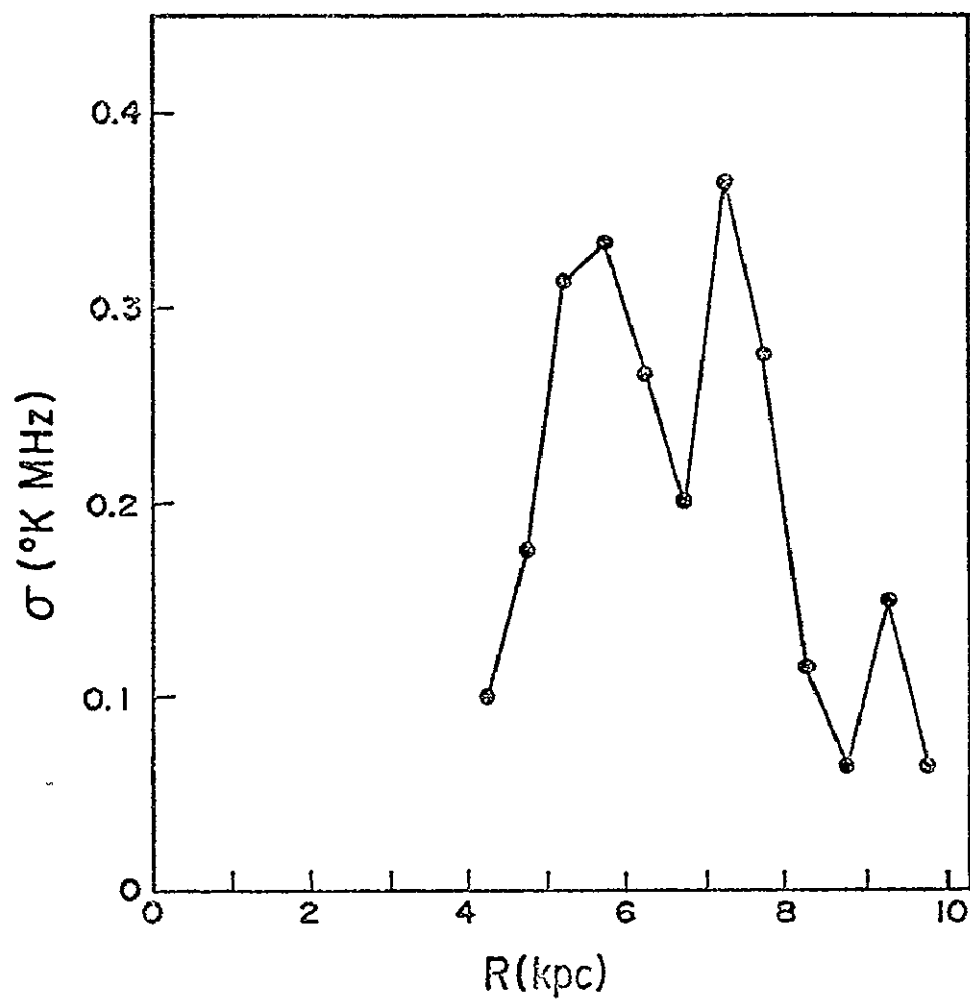


Figure V-3. CO surface brightness of the Galaxy as would be seen by a distant observer viewing the Galaxy face-on.

and velocities (Fig. IV-1), and several numerical tests all suggest that such a displacement exists over a wide range of longitudes, demonstrating that this effect is a general galactic phenomenon rather than the result of a single large complex south of the galactic equator.

A displacement of the plane of this magnitude is consistent with the position of the HI plane in the region surveyed. Examination of a typical 21 cm latitude-velocity contour plot (Figure V-4) reveals that the position of the emission centroid with respect to $b = 0^\circ$ fluctuates on the scale of the CO plane depression (roughly $0^\circ.3$). Quigora (1974) and Henderson (1967) have actually measured the position of the HI layer and have found that it varies about ± 50 pc inside the solar radius.

There has also been a rough determination of the position of the SNR layer made by Ilovitsky and Lequeux (1972). When the SNR and CO results are plotted together (Fig. V-5), a rough correlation is apparent. Lockman (1977) has determined that the HII regions also generally lie at negative z . So once again it appears that CO acts as a good tracer of the early Population I objects.

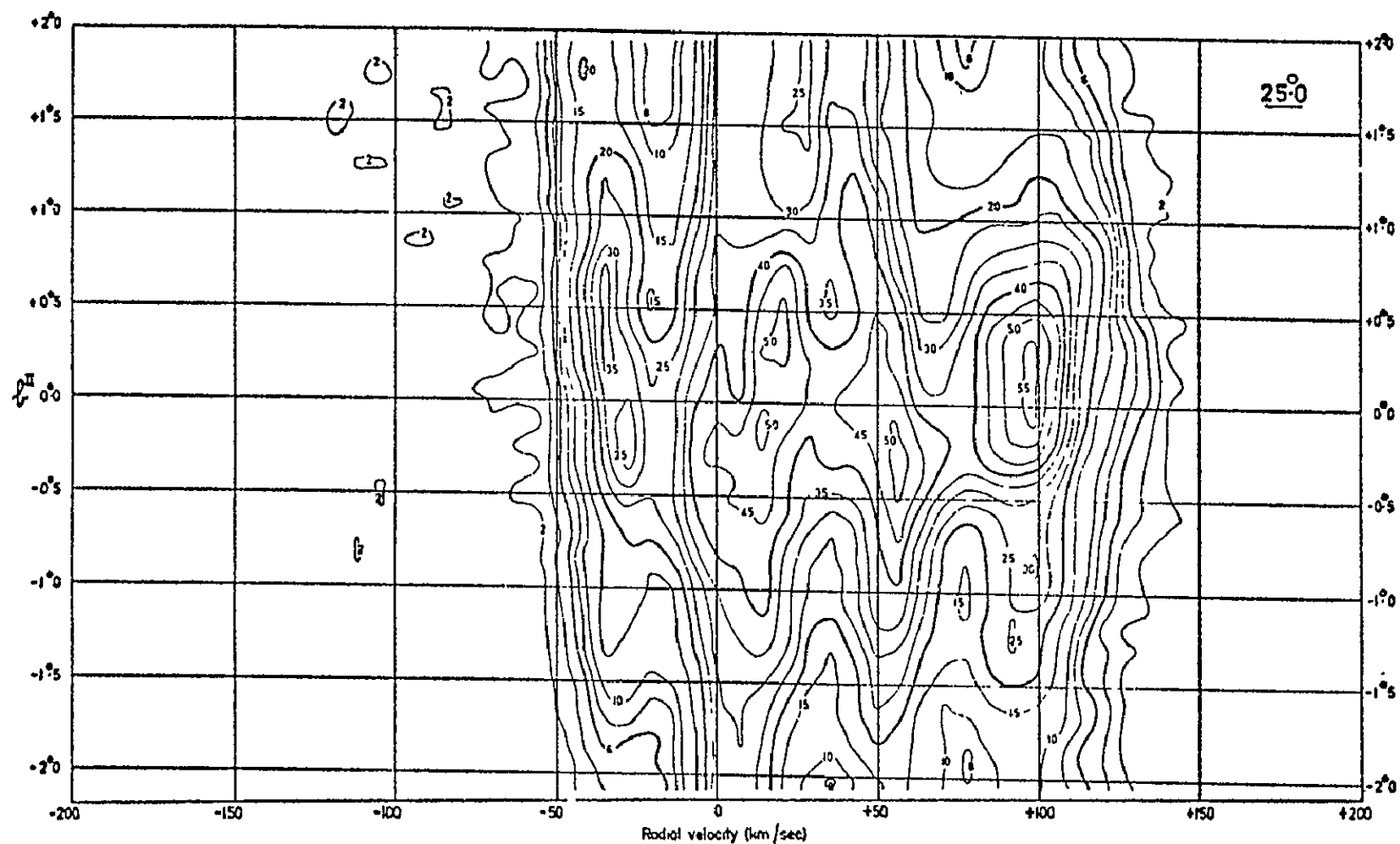


Figure V-4. Distribution of HI emission at $l = 25^\circ$ as a function of v_{LSR} and galactic latitude, from Kerr (1969). Note that the latitude of the galactic "plane" fluctuates by about 0.3° .

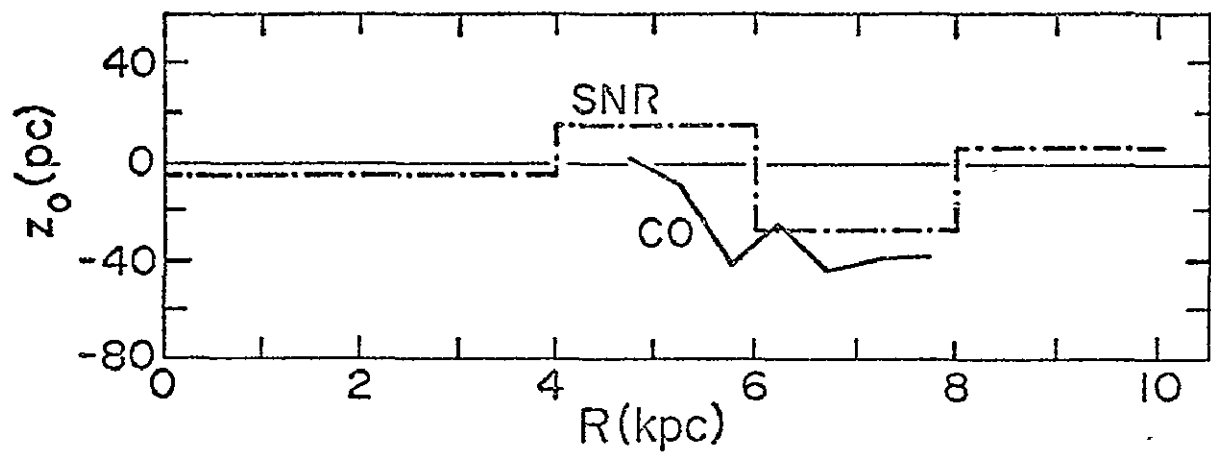


Figure V-5. Displacement of the galactic disk from the conventional galactic plane as determined using CO (present work) and supernova remnants (Ilovaisky and Lequeux, 1972).

Comparison with γ -ray data

Finally, we consider the γ -ray data and its relationship to CO. As we have seen, the similarity of the γ -ray and CO radial distributions implies that most of the interstellar gas is molecular, but, because of the uncertainty of the cosmic-ray distribution, the argument is not conclusive. However, if, as seems likely, the cosmic-ray disk is substantially thicker than the gaseous disk, then the thickness of the γ -ray disk would be an excellent measure of the thickness of the interstellar mass. Because the atomic disk is twice as thick as the molecular, the γ -ray thickness observations, when they become available, will give a good indication of the relative molecular and atomic masses.

Summary

To sum up, a new out-of-plane survey of CO line emission in the first quadrant of the Galaxy has confirmed the findings of previous in-plane surveys that CO is concentrated in a molecular ring about 6 kpc from the galactic center, and has furnished determinations of two other important properties of the molecular disk as functions of galactocentric distance, namely the thickness of the disk, and its displacement with respect to the currently adopted galactic equator.

Comparison with other constituents of the Galaxy has confirmed that CO can be used as a tracer of young Populations I objects and is, in fact, because of its ubiquity, the best available tool for observations of this component. On the other hand, HI has a different distribution, both in z and in radius, but its thickness does increase at roughly the same galactic radius as that of CO.

This thesis has reported on the first phase of an on-going systematic survey of the galactic disk. As additional data becomes available, it should be possible to relax some of the constraints of the model, particularly azimuthal symmetry, and produce a better picture of the distribution of molecular matter. Also, more detailed studies of individual molecular complexes, combined with higher resolution studies of individual distant clouds done with larger antennas, will elucidate the nature of the molecular regions..

APPENDIX A

Telescope Pointing

Radio telescopes, unlike most optical telescopes, normally operate without constant monitoring of their position with respect to known astronomical objects. An accurate and dependable method for determining the position of the telescope is therefore essential. In the case of the 4-foot telescope, the pointing is further complicated by the absence of astronomical point sources with sufficient flux at the operating frequency to be detected with a 4-foot telescope, and by a desire for a high degree of automation of the telescope drive.

These problems were solved by using accurate computer readable position encoders on the telescope axes, calibrating the pointing using an optical telescope attached to the radio dish, and assuring the colinearity of the radio and optical axes. In addition, frequent direct radio frequency checks were made against the position of the sun.

Shaft encoders:

The position of the telescope is determined by Baldwin 15 bit optical shaft encoders mounted on each axis. These encoders give a resolution of $1/2^{15}$ revolution or 39.6 arc seconds (referred to below as one bit). The encoder accuracy

was assured by two testing procedures, one sensitive to high angular frequency errors, and the other to low frequency errors.

The low frequency errors, which would produce systematic pointing errors as a function of sky position, were measured by comparing the encoders with the precisely machined worm wheels that were part of the telescope drive. The encoders were mounted on the telescope (with the antenna dish and front end removed). The axis to be tested was then rotated under computer control in approximately 1/1000 revolution units using the stepping motor that is part of the normal drive system. At each position the shaft encoder was read by the computer and compared with the reading that would be expected if the worm wheel and encoder were both perfect. Figures A-1 and A-2 show the difference between the actual and expected readings as a function of rotation angle for a complete revolution. Note that because 1/1000 revolution corresponds to about one half revolution of the stepping motors (which have negligible cumulative errors over many revolutions) inaccuracies in the position of the motor steps would appear as a very high frequency component in the results.

Figure A-1 shows the results of the test of the altitude encoder. The start of both graphs corresponds to

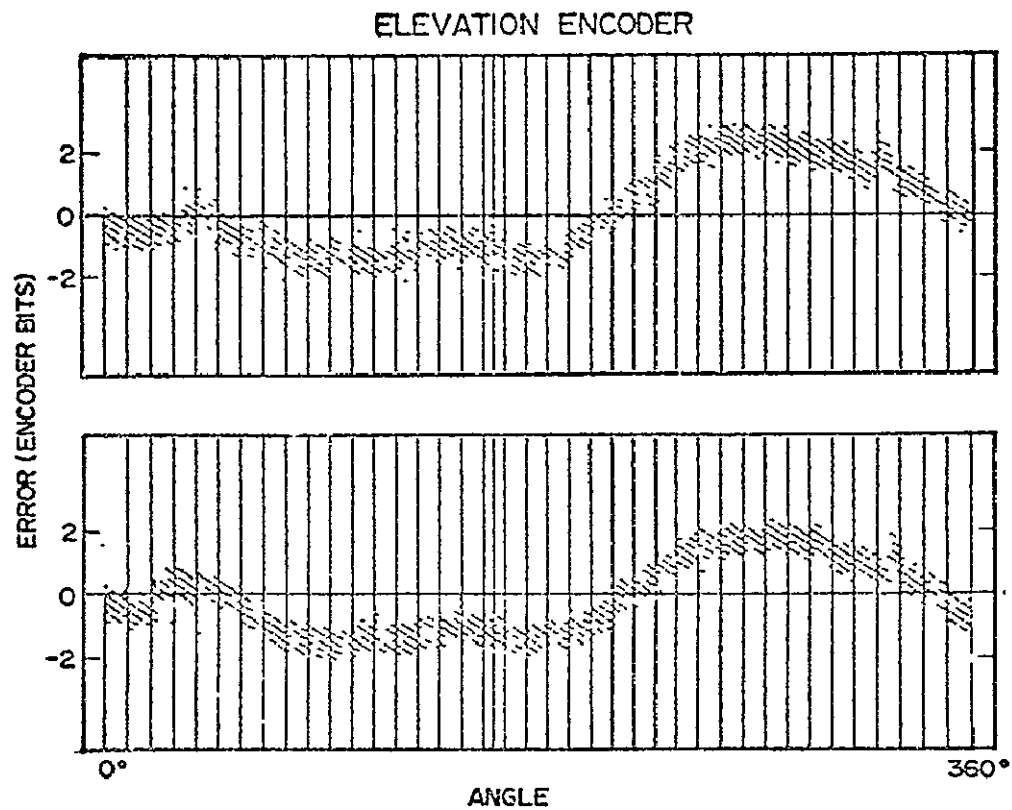


Figure A-1. Top: Deviation of elevation shaft encoder reading from expected value vs position of mount.

Bottom: Same, except encoder rotated 180° with respect to the mount.

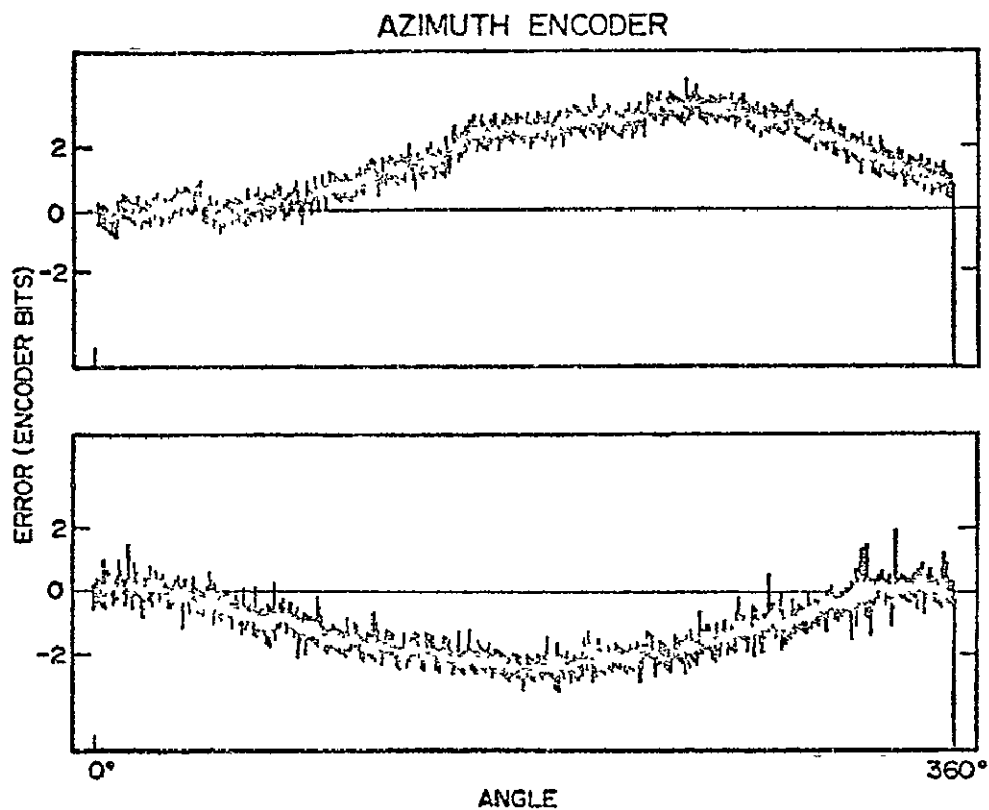


Figure A-2. Top: Deviation of azimuth shaft encoder reading from expected value vs position of mount. Bottom: Same, except encoder rotated 180° with respect to the mount.

approximately the same position of the telescope mount, but different positions of the encoder, which was rotated 180° with respect to the mount between these tests. Most of the structure in the plots is the same before and after the encoder rotation and is therefore associated with the gear on the mount rather than with the encoder.

Figure A-2 shows the results of tests on the azimuth axis, with the encoder rotated 180° between tests. The source of the shape on these plots could not be established. On both axes the peak to peak error is 5 bits of which one bit is due to roundoff effects. The system error is thus less than ± 2 bits. This error is a combined error due to encoder and mount and thus is an upper bound on the encoder accuracy.

To test for high frequency errors, the encoder was securely mounted in a test jig and the shaft was rotated at approximately 1 RPM by a DC motor powered by a regulated supply. The encoder was then read at equal time intervals of a few milliseconds by the computer and the number of times each encoder value occurred was tallied. In an ideal system each value would occur the same number of times; deviations from this ideal are the result of some bit patterns occurring over larger angular widths than others. Figure A-3 shows the

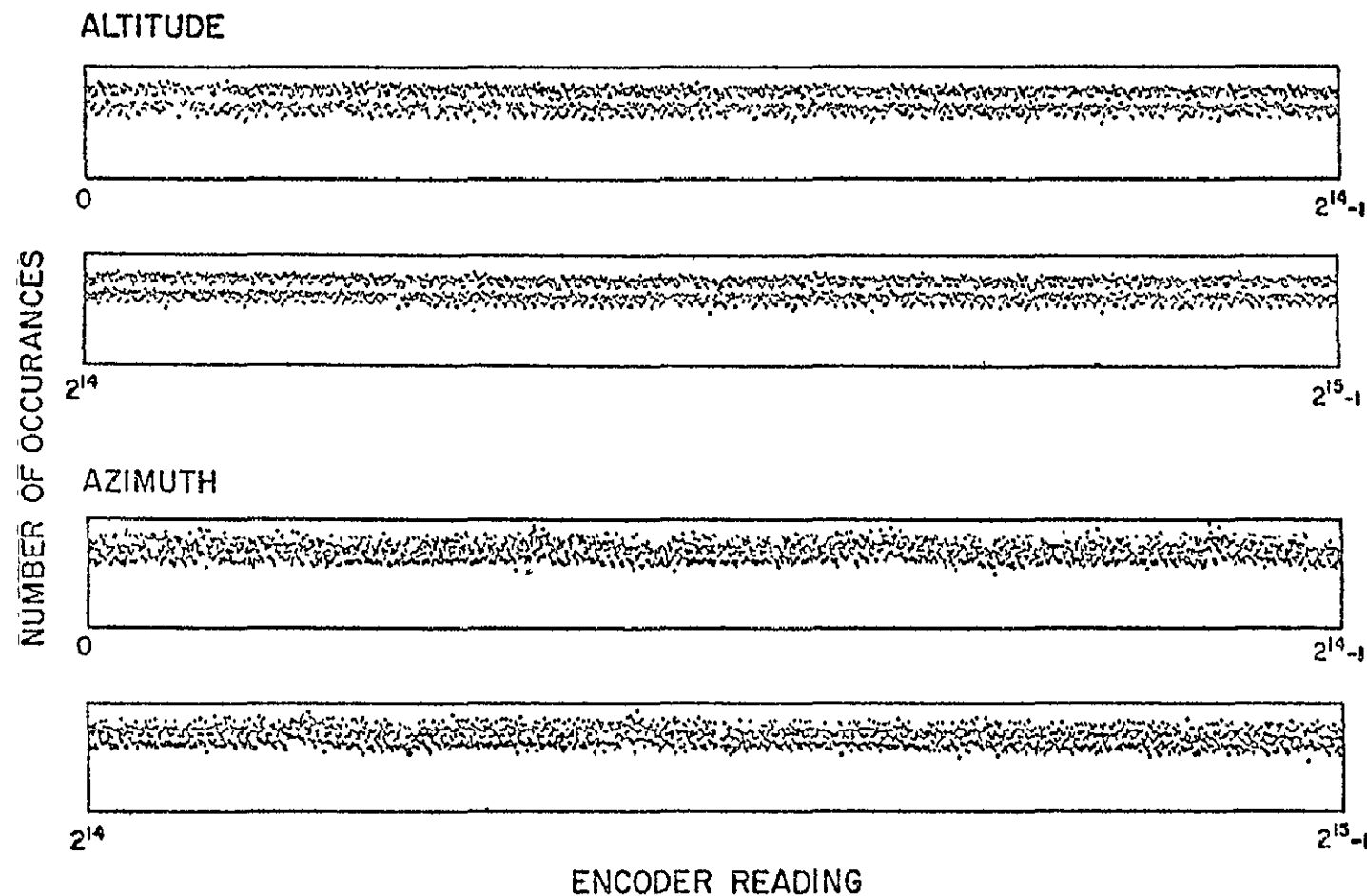


Figure A-3. Test of shaft encoders. Number of occurrences of each possible encoder reading during test. Ideally, each reading would have occurred the same number of times.

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number of times each value occurred. Histograms of the number of values that occurred with a given frequency are shown in figure A-4. From these figures it is clear that the deviation in the angular sizes corresponding to individual encoder values is about 1/3 bit.

Pointing model and initial determination of the properties of the mount:

The azimuth axis was first set to the vertical to within 0.5 arc minutes with a precision bubble level. Then, using the optical telescope mounted on the radio dish, an initial determination of the properties of the mount was made. Following Chauvenet (1960), the pointing of the optical telescope was modeled by the following equations:

$$A = A' + \Delta A + b \cot(z' + \Delta z) + c \csc(z' + \Delta z) + 180^\circ \theta(-z' - \Delta z) \quad (A.1)$$

$$z = |z' + \Delta z| - R \quad (A.2)$$

$$\phi = \phi' + \Delta\phi \quad (A.3)$$

$$\lambda = \lambda' + \Delta\lambda \quad (A.4)$$

where A = true azimuth

A' = azimuth encoder reading

b = correction due to non-perpendicularity of altitude and azimuth axes

c = correction due to non-perpendicularity of altitude

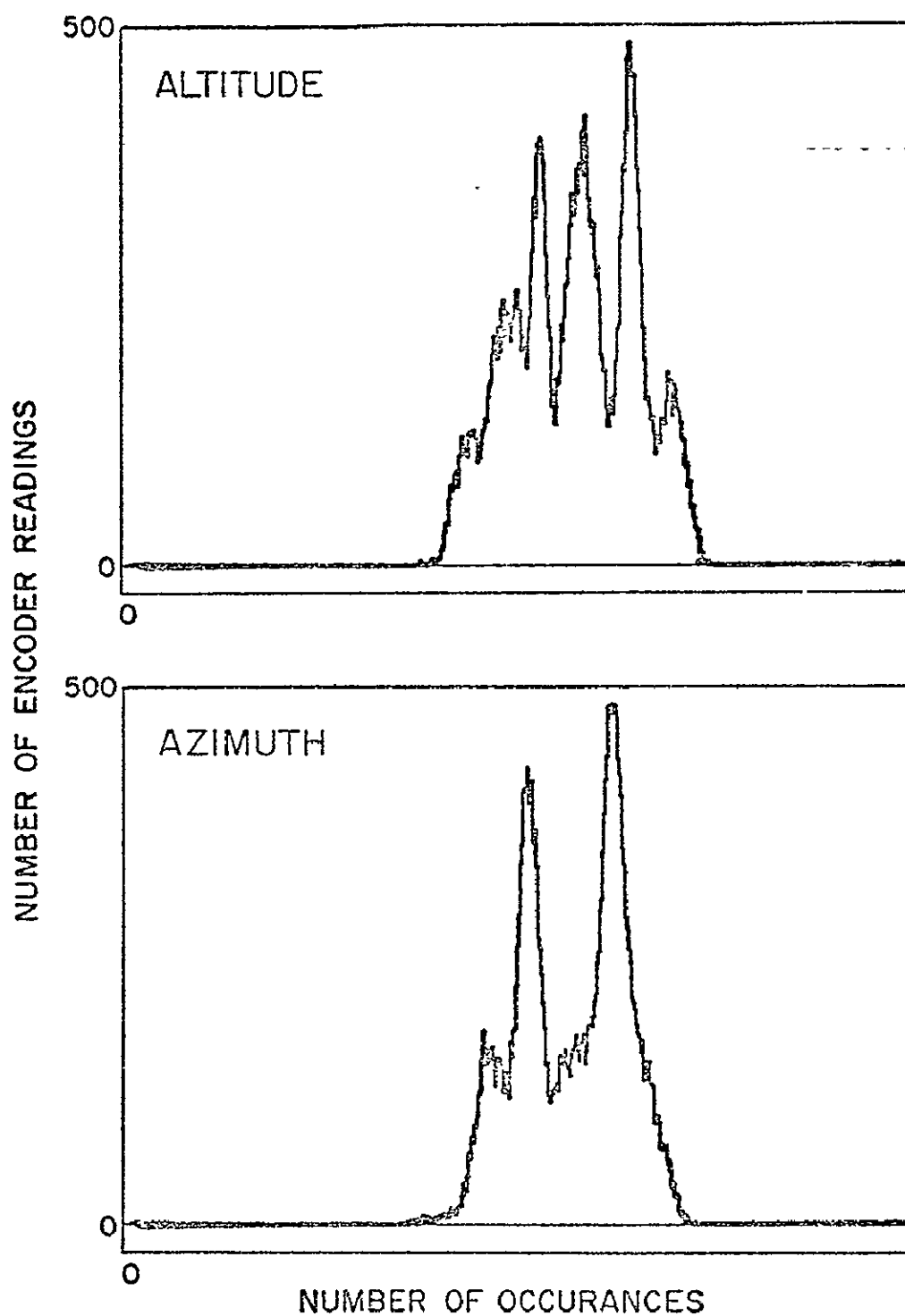


Figure A-4. Data of Figure A-3 histogrammed by number of occurrences. Ideally, each encoder reading would have occurred the same number of times and the histogram would be zero except at a single point.

axis and optical axis

z = true zenith angle = 90° - altitude

z' = zenith angle encoder reading

ϕ = apparent latitude (to be used in right ascension
declination to hour angle, altitude transformation)

ϕ' = true latitude of observatory = 40.8101°

λ = apparent longitude

λ' = true longitude of observatory = 73.9617°

$R = 1 \text{ arcmin } \tan z$ = correction for refraction

$\theta(x) = 1 \text{ for } x > 0, 0 \text{ for } x < 0$

and ΔA , Δz , $\Delta\phi$, and $\Delta\lambda$ are offsets.

ϕ and λ depend on the true position of the vertical (azimuth) axis, and ΔA and Δz depend on the mounting of the encoders. The absolute value in the expression for z and the θ -function are required because two different telescope positions correspond to the same sky position.

Approximate values of ΔA and Δz (the only large correction terms) are obtained by sighting on a point (the top of the Palisades toward Alpine Tower) whose coordinates relative to the telescope had been determined from a map ($A = 10.76^\circ$, $90-z = 0.17^\circ$).

The value of c can be determined by sighting any position on the horizon ($z = 90^\circ$) in both positions of the tele-

scope. In the first case equation A.1 reduces to

$$A = A' + \Delta A + c$$

and in other positions we get

$$A = A'' + 180^\circ + \Delta A - c$$

where A' and A'' are the apparent positions in the two positions. Solving for c we get

$$c = (A'' - A' + 180^\circ)/2.$$

Similarly, by observing the pole star in both positions of the mount, all the other parameters except λ can be determined. Denoting the azimuth encoder reading, zenith angle and time of measurement in the first position by A' , z' and t' in the second position by A'' , z'' , and t'' equation A.1 give to first order in b , c , and R .

$$A_1 = A' + \Delta A + b \cot z + c \csc z$$

$$A_2 = A'' + \Delta A - b \cot z - c \csc z + 180^\circ$$

where A_1 and A_2 the true azimuths of the pole star at times t' and t'' as computed from the American Ephemeris and Nautical Almanac (AENA) pole star table. Approximating $z=90^\circ-\phi$ and solving for b and ΔA we get

$$b = \frac{1}{2}(A_1 - A_2) \cot \phi - c \csc \phi + \frac{1}{2}(A'' - A' + 180^\circ) \cot \phi$$

$$\Delta A = \frac{1}{2}(A_1 + A_2 - A' - A'' - 180^\circ).$$

Using the constants a_0' , a_0'' (for times t' and t''), a_1 , and a_2 given in the AENA pole star table, the true zenith angle at each position can be expressed as

$$z' + \Delta z - R = \phi - a_0' - a_1 - a_2$$

$$\begin{aligned} |z'' + \Delta z| - R &= -z'' - \Delta z - R \\ &= \phi - a_0'' - a_1 - a_2 \end{aligned}$$

Solving for z and ϕ we get

$$\Delta z = \frac{1}{2}[(-z'') - z' + a_0'' - a_0']$$

$$\phi = \frac{1}{2}[(-z'') + z' + a_0'' + a_0'] + a_1 + a_2 - R$$

As a result of the procedures described above, the values of ϕ , ΔA , and Δz were determined to sufficient accuracy that it was possible to easily find stars in the optical telescope. Since the value of b , the non-perpendicularity of the mount axes, was less than $1'$ of arc, it was assumed that it could be ignored. This hypothesis was later verified more accurately by the star pointing measurements.

Alignment of optical and radio axes:

Using radio measurements, the only position that can be easily located with the 4-foot telescope to high accuracy is the center of the sun. Because the limb of the sun is extremely sharp at millimeter wavelengths, it was easy to place the radio axis of the telescope on the center of the

sun by measuring the position of several points on the limb. With the telescope tracking the center of the sun on its radio axis, the axis of the optical telescope was adjusted until its crosshairs also lay at the center of the sun. This was done to a fraction of an arc minute by focusing the solar image on a circular "target" (a piece of polar coordinate graph paper will do) the same size as the solar image and adjusting the optical telescope until the sun was centered on the target and the image of the crosshairs was at the center of the target.

Star pointing:

With the radio and optical axes aligned and approximate parameters determined, the model of equations A.1-A.4 with $b = c = 0$ was used to point the telescope, under computer control, at a number of stars scattered throughout the sky. After a star was located in the optical telescope, the telescope was moved until the star was exactly centered on the optical crosshairs and the apparent star position was recorded. By varying the parameters of the model to minimize the least-squares deviation between the coordinates given by the model and the true positions of the stars, improved model parameters were found. It was determined that b and c could be taken to be zero without introducing any

significant error.

With the improved values of ΔA , Δz , $\Delta \lambda$ and $\Delta \phi$ inserted into the model, the deviation of each star was again computed and the RMS deviation was calculated. The final test included 31 stars. The maximum error at any star was 2.2' and the RMS error was 1.1'. Figures II-2 and II-3 show the residual errors as functions of azimuth and elevation.

Solar pointing:

Additional frequent checks of the pointing were made against the position of the center of the sun using radio measurements. An automatic computer program scanned the telescope across the sun in azimuth and elevation, computed the position of the center of the sun by finding the points of maximum symmetry of the intensities measured along each axis, and computed the error from the expected (true) solar position. These checks gave peak errors of 2' and typical errors of about 1.5'.

APPENDIX B

Antenna Pattern

The antenna was designed to be diffraction limited. Before delivery the manufacturer measured the surface of the main reflector and found a peak error of 0.0014 inch. This corresponds to $\lambda/70$ at 2.6 mm, much better than the usual engineering standard of $\lambda/40$ for diffraction limited systems.

In order to verify that the dish actually met this specification and also to determine the tolerances permitted in positioning the feed horn, a series of calculations were made and were compared with actual measurements of the antenna pattern. The telescope was tested at 115 GHz against a transmitter across the Columbia campus at a distance of 420 meters corresponding to $.72 D^2/\lambda$. In antenna testing, the transmitter is conventionally placed at a distance at least $2D^2/\lambda$ from the antenna; however, Johnson et al. have shown that patterns very similar to the far field patterns can be obtained at as little as $D^2/8\lambda$ if the telescope is refocused on the near field transmitter (Johnson et al. 1973).

The theoretical antenna patterns were calculated using scalar Kirchhoff diffraction theory. All the calculations described here are for the "equivalent parabola" (Hannan 1961), the locus of the intersections of the incoming rays with the extensions of the corresponding rays reflected from

the secondary (see figure B-1). This equivalent parabola is the same diameter as the actual primary reflector and has the same focal length as the Cassegrain system. Near the focus, the pattern and focusing properties of this single parabola are very close to those of the original Cassegrain parabola and secondary.

The field of an arbitrary aperture in the Fraunhofer region is given by (Silver 1964):

$$U(\theta, \phi) = (i/\lambda R) e^{-ikR} g(\theta, \phi)$$

$$\text{where } g(\theta, \phi) = \int_{\text{aperture}} F(x, y) e^{i(k_x x + k_y y)} dx dy$$

$$k_x = k \sin \theta \cos \phi$$

$$k_y = k \sin \theta \sin \phi$$

$$k = 2\pi/\lambda$$

and $F(x, y)$ is a complex function which gives the field over the aperture.

$F(x, y)$ is approximated in this calculation by zero outside the dish and on the blocked portion of the aperture and elsewhere by*

$$F(x, y) = F(\rho) = (1 - (\rho/a)^2) \times (1 - E) + E$$

where E is the ratio of the field at the edge of the dish

*see section on "Horn Pattern"

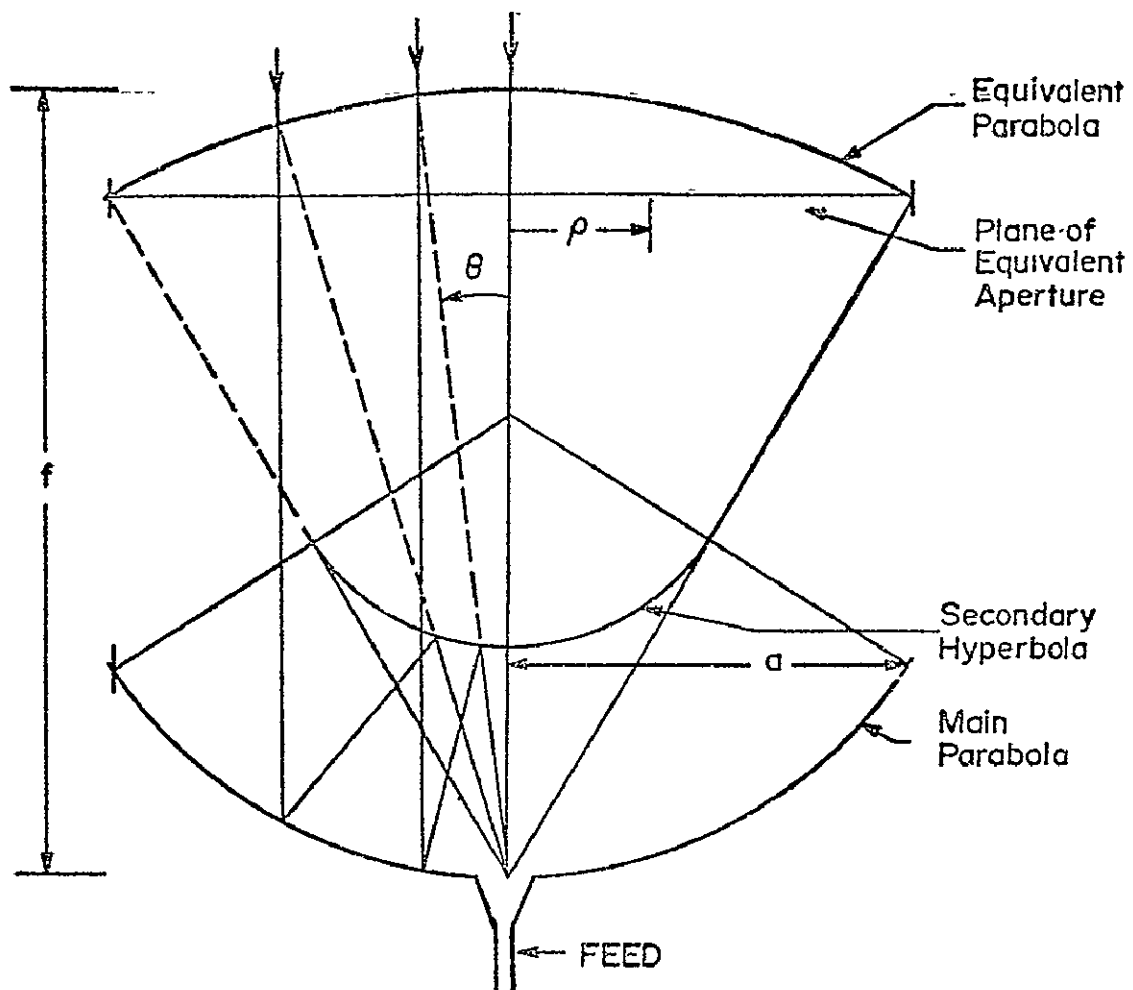


Figure B-1. Definition of equivalent parabola of focal length f and radius a . ρ is the radial coordinate in the plane of the equivalent parabola.

to the field at the center. Then

$$g(\theta, \phi) = g_o(\theta, \phi) - g_B(0, \phi)$$

where

$$g_o = \int_0^{2\pi} d\phi' \int_0^a d\rho \rho F(\rho) e^{ik\rho \sin\theta \cos(\phi - \phi')}$$

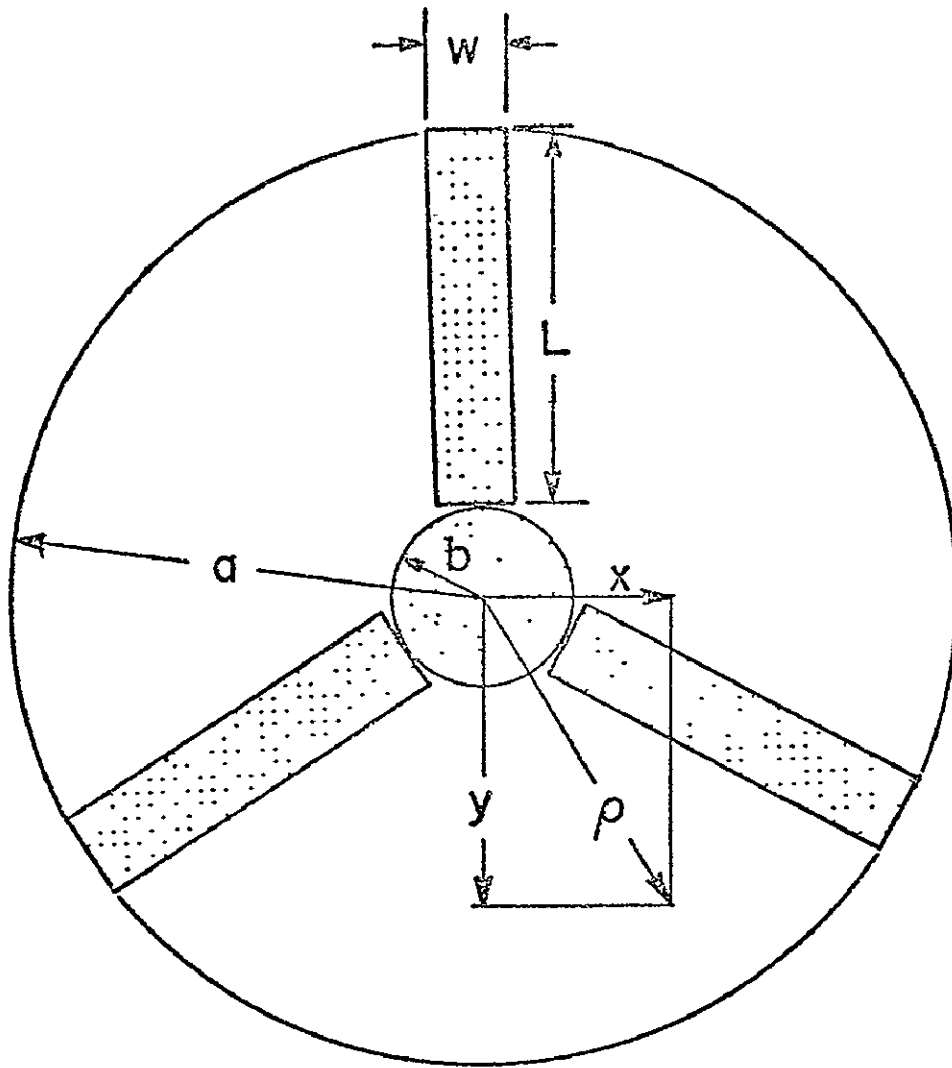
is the field of the unblocked aperture and g_B has the same integrand but is integrated over the blockage. The blockage is approximated by a central circle representing the secondary and three rectangles representing the arms (see figure B-2). Thus g_B is given by $g_B = g_C + g_A$ where g_C is an integral over the central blockage and g_A is an integral over the arms.

Using these approximations the integrals can all be done analytically. Letting $r = \rho/a$, $u_x = \pi x \sin\theta/\lambda$ and $f(r) = F(ra)$ we get

$$g_o = 2\pi a^2 \left[\frac{J_1(u_{2a})}{u_{2a}} E + \frac{J_2(u_{2a})}{u_{2a}^2} (1-E) \right]$$

$$g_C(\theta, \phi) = 2\pi b^2 \left[\frac{J_1(u_{2b})}{u_{2b}} \left[E + (1 - (b/a)^2)(1-E) \right] + \frac{J_2(u_{2b})}{u_{2b}^2} (b/a)^2 (1-E) \right]$$

$$g_A(\theta, \phi) = \sum_{\alpha=0, \frac{2\pi}{3}, \frac{4\pi}{3}} \left[ELW \frac{\sin(u_L \cos(\phi - \alpha))}{u_L \cos(\phi - \alpha)} \frac{\sin(u_W \sin(\phi - \alpha))}{u_W \sin(\phi - \alpha)} \right. \\ \left. \times \exp(i 2 u_{b+L/2} \cos(\phi - \alpha)) \right. \\ \left. + (1-E) W a \frac{\sin(u_W \sin(\phi - \alpha))}{u_W \sin(\phi - \alpha)} \int_{b/a}^1 (1-r^2) \exp[i 2 u_a \cos(\phi - \alpha) r] dr \right]$$



$$\begin{aligned}
 b &= 7.6 \text{ cm} \\
 a &= 61.0 \text{ cm} \\
 w &= 2.5 \text{ cm} \\
 L &= 53.3 \text{ cm}
 \end{aligned}$$

Figure B-2. Aperture and blockage used in calculation of antenna pattern. The circle of radius a is the aperture and the shaded areas are blockages. x , y , and ρ define the coordinate system used in the calculation.

The pattern intensities are proportional to $|g|^2$. Figure B-3 shows the intensities in the altitude and azimuth planes normalized to 0 dB at the central maximum for 10 dB edge illumination. These graphs also show the pattern that would be obtained with an unblocked circular aperture and with only the central blockage. Note that without the blockage the pattern is azimuthally symmetric in this approximation. Figures B-4, -5, -6, and -7 show several sets of measured points along with the calculated pattern.

Horn Pattern

Figure B-8 shows the dimensions of the two available horns. Since the phase errors across the mouths of these horns are small ($< \lambda/6$), the horns can be modeled to good precision by a rectangular waveguide antenna. The field in the central portion of the pattern is (Silver, 1964):

$$E_{\theta} \sim \frac{\sin u_b}{u_b} = g_E(u_b)$$

$$E_{\phi} \sim \frac{\pi^2}{4} \frac{\cos u_a}{u_a^2 - \pi^2/4} \equiv g_H(u_a)$$

where $u_x = \frac{\pi x}{\lambda} \sin \theta$

C-2

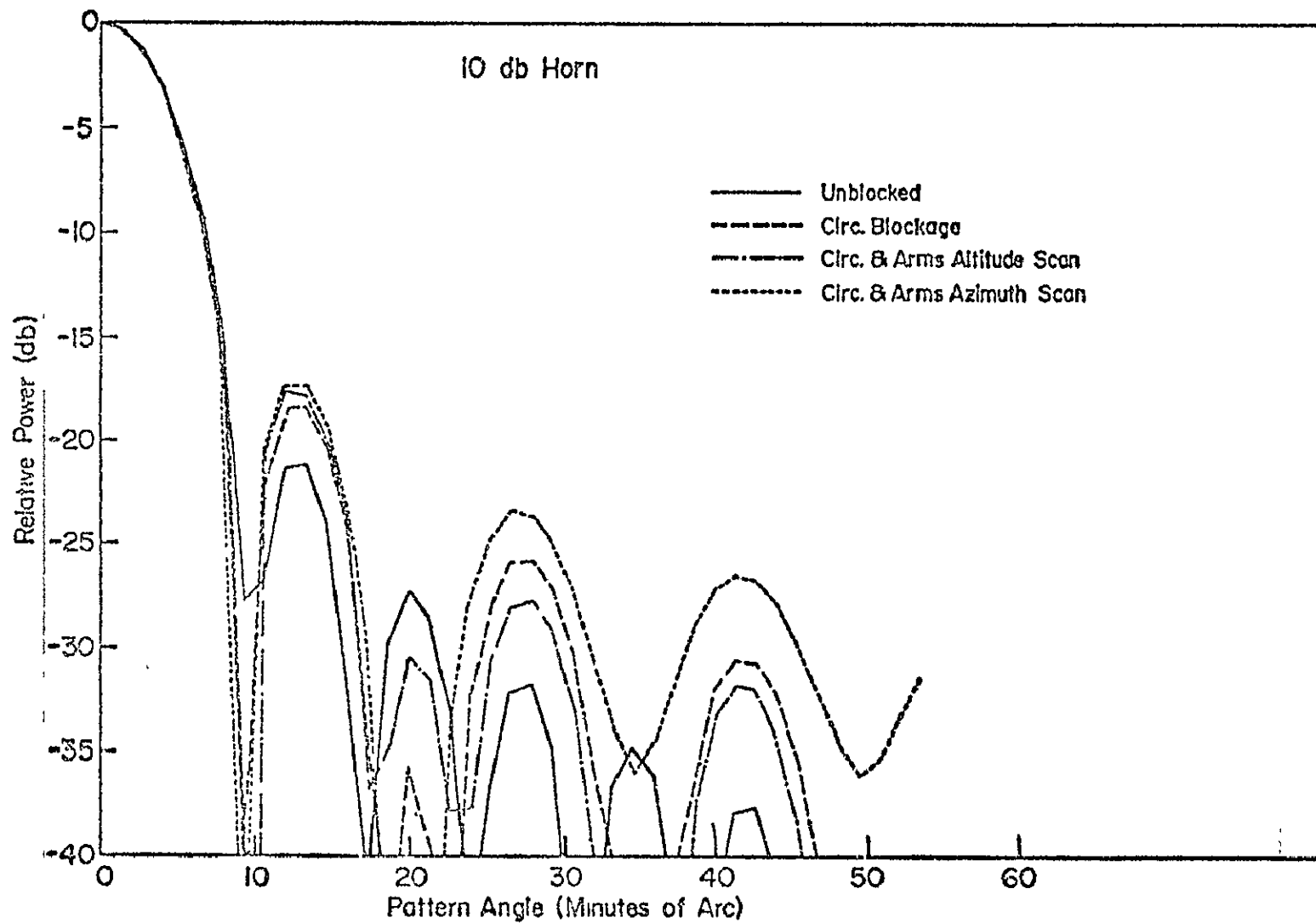


Figure B-3. Calculated antenna pattern with 10 dB edge illumination.

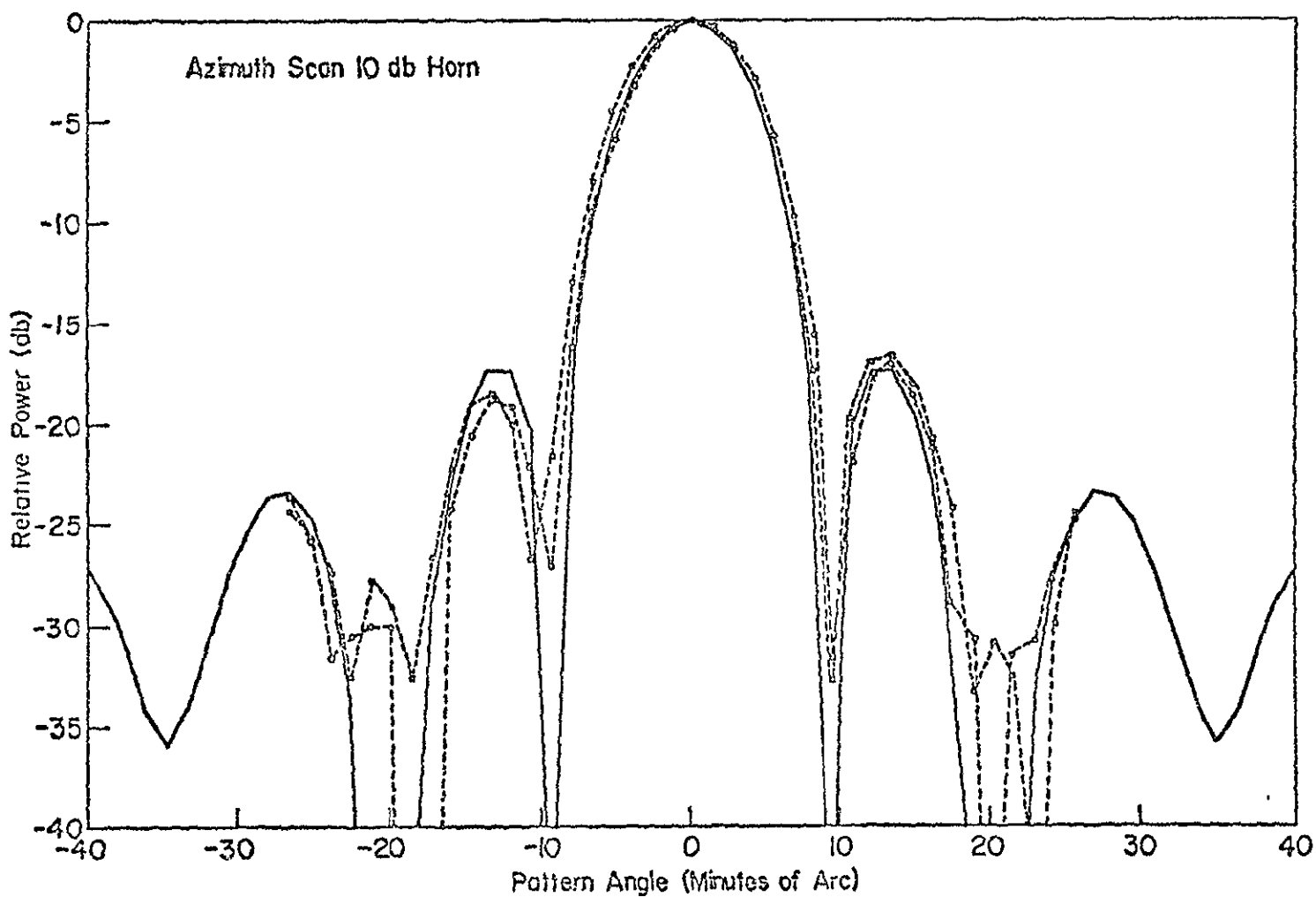


Figure B-4. Antenna pattern in horizontal plane with 10 dB edge illumination. Solid lines are the calculated patterns. Dotted lines are the measurements.

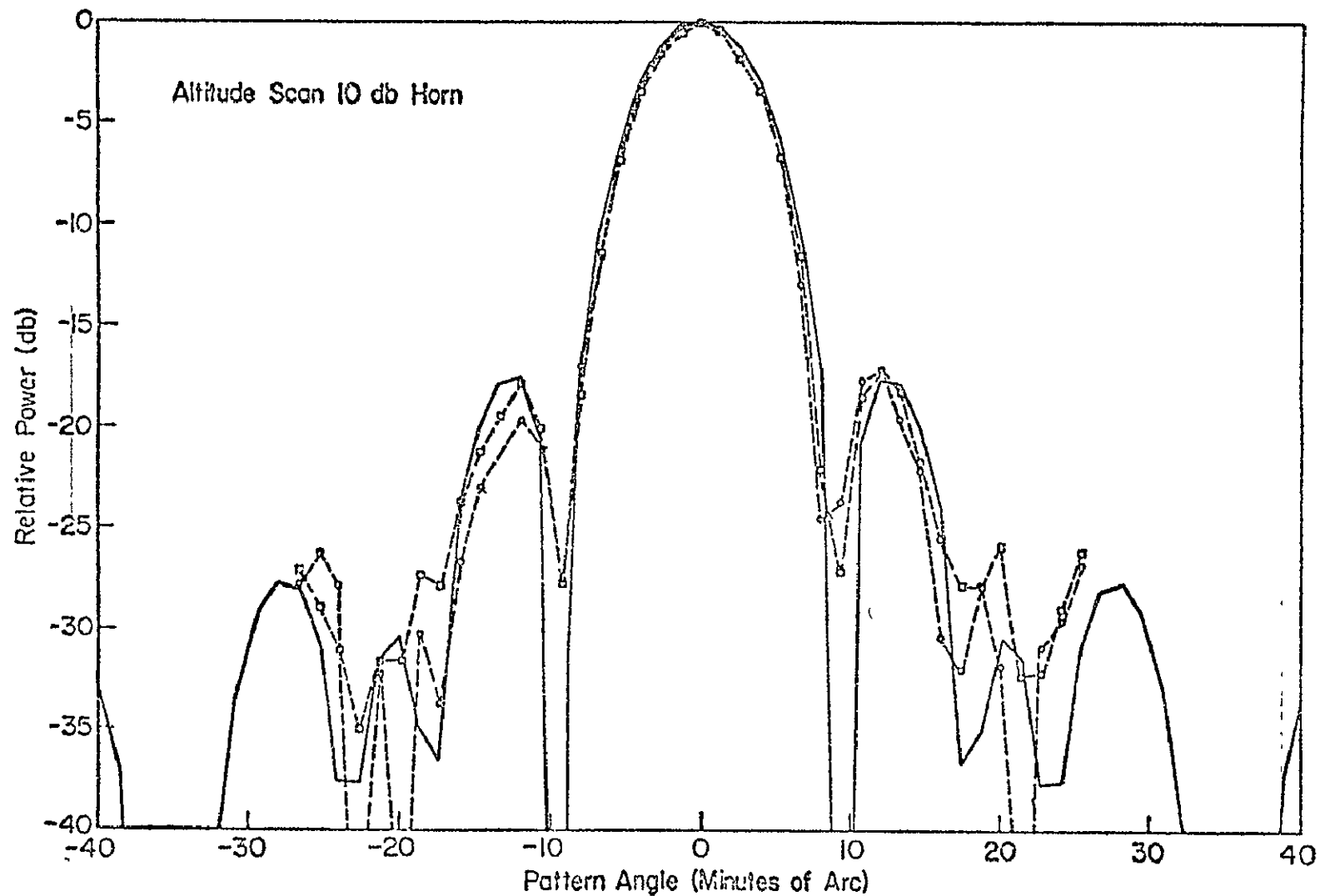
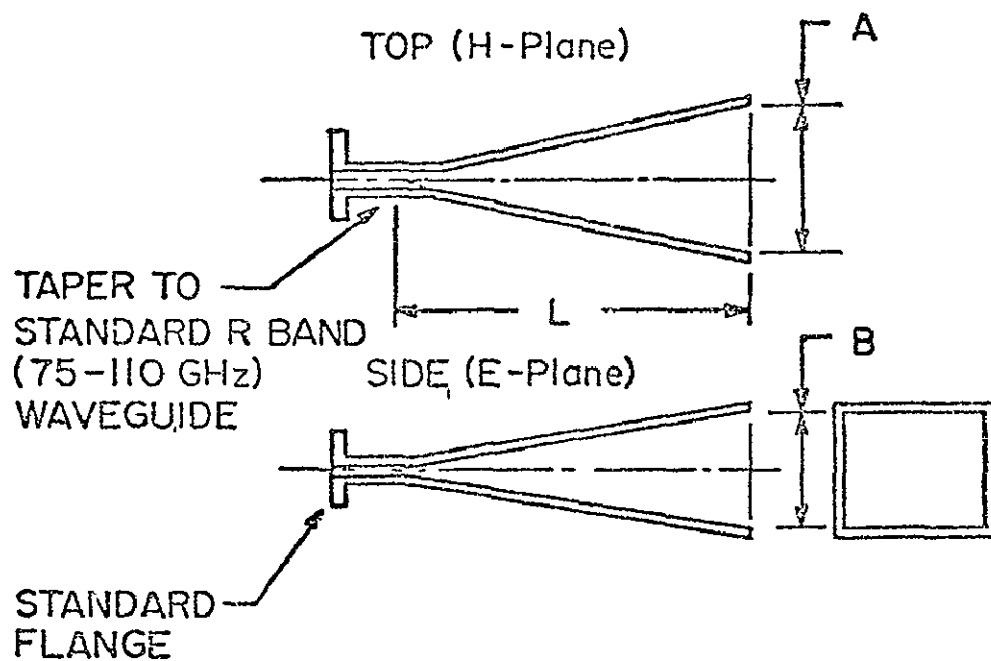


Figure B-5. Antenna pattern in vertical plane with 10 dB edge illumination. Solid lines are the calculated patterns. Dotted lines are the measurements.



	10 db HORN	13 db HORN
A	0.618 ± 0.003	0.715 ± 0.003
B	0.431 ± 0.003	0.486 ± 0.003
L	3.50 ± 0.03	3.50 ± 0.03

Figure B-6./ Dimensions of the two available feed horns.

and E_θ and E_ϕ are the E-fields in the E and H planes of the waveguide.

For computational purposes the field was approximated by

$$g(\rho) = E + (1-E) \times [1 - (\rho/a)^2]$$

where E is the field at the edge of the main dish. Figure B-7 shows the aperture illumination functions g_E , g_H , and g for the 10 dB horn. All these curves lie within 1.5 dB of each other.

Feed Horn Positioning

A.) On-axis defocusing.

The predominant effect is a phase error across the aperture given by

$$\Psi = 2\pi \frac{d}{\lambda} (1 - \cos\theta)$$

where d is the amount of defocusing and θ is the aperture angle in the equivalent parabola (see Fig. B-1). For a long focal length system $\theta \ll 1$, thus

$$\Psi \approx \frac{2\pi d}{\lambda} \frac{\theta^2}{2}$$

$$\Psi_{\max} \approx \frac{\pi d}{\lambda} \theta_{\max}^2 \approx \frac{\pi d}{\lambda} \left(\frac{D}{2f} \right)^2 = \frac{\pi d}{4\lambda} \left(\frac{D}{f} \right)^2$$

If we limit Ψ to $\Psi_{\max} < \pi/4$, the effect of the phase error will be small. Thus we must have

$$d < \frac{4\lambda}{\pi} \left(\frac{f}{D} \right)^2 \times \left(\frac{\pi}{4} \right) = 0.78 \text{ inches}$$

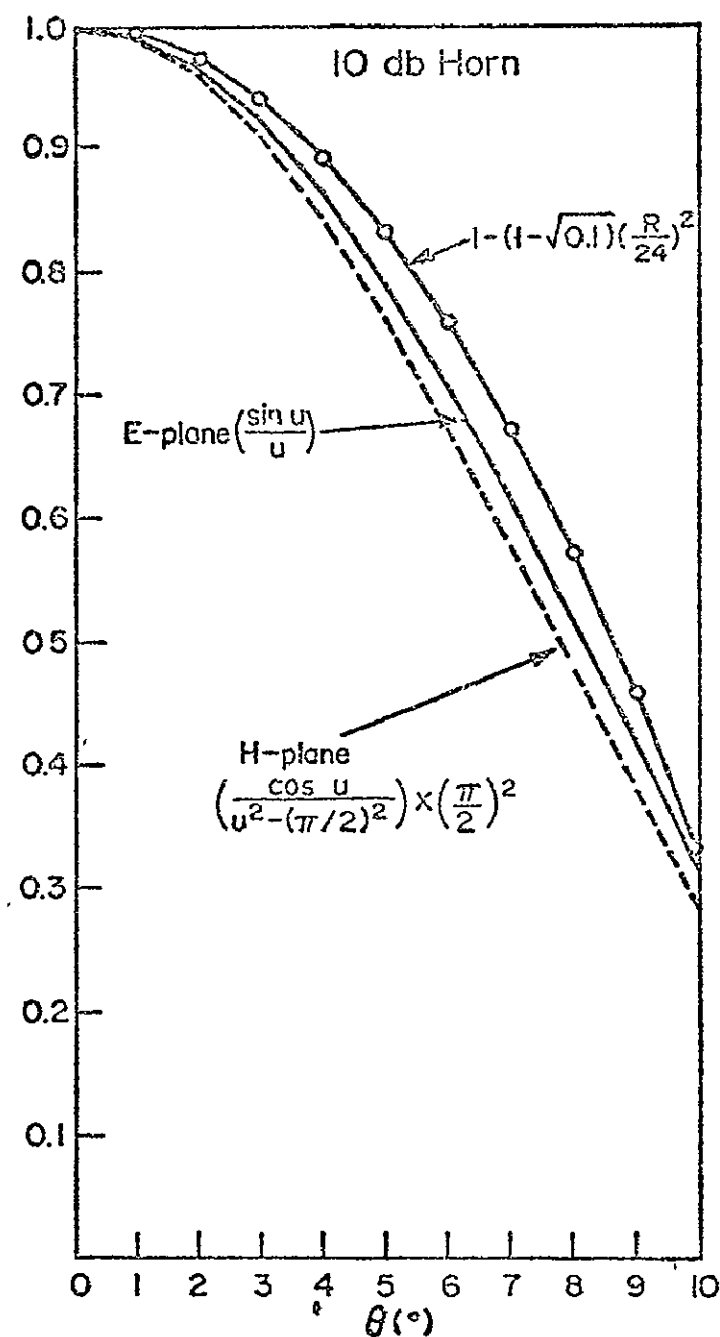


Figure B-7. Aperture illumination functions for the 10 dB horn in the E- and H-planes and the approximate function used in the calculations.

Experiment confirms that it is adequate to focus to $\pm 3/4$ inch (see Figure B-10).

B.) Off-axis defocusing

The result of moving the horn off axis is well covered in the literature (Ruze, 1965; Sandler, 1960; Kelleher, 1952; Lo, 1960). The major effect is a shift in the position of the primary lobe of the antenna system. If, in the equivalent single reflector antenna, the horn is shifted an angle α with respect to the center of the parabola, the beam will shift by $\beta = B\alpha$ where B is known as the "beam deviation factor" and is a function of f/D . For an $f/2.8$ system, we can take $B = 1$ to high accuracy.

In addition, when the offset becomes large, there is an asymmetric coma distortion of the antenna pattern. Ruze (1965) shows that there is negligible (<1 dB) change in the pattern shape if $\alpha < \frac{\lambda}{D} 2 \left(\frac{f}{D} \right)^2$. If the horn is shifted by a distance d ($d \ll f$) then $\alpha \approx d/f$. The criterion is then $d < 2\lambda \left(\frac{f}{D} \right)^3 \approx 4.2$ inches. Since the feed is easily positioned to this accuracy, no distortions of the pattern should be seen.

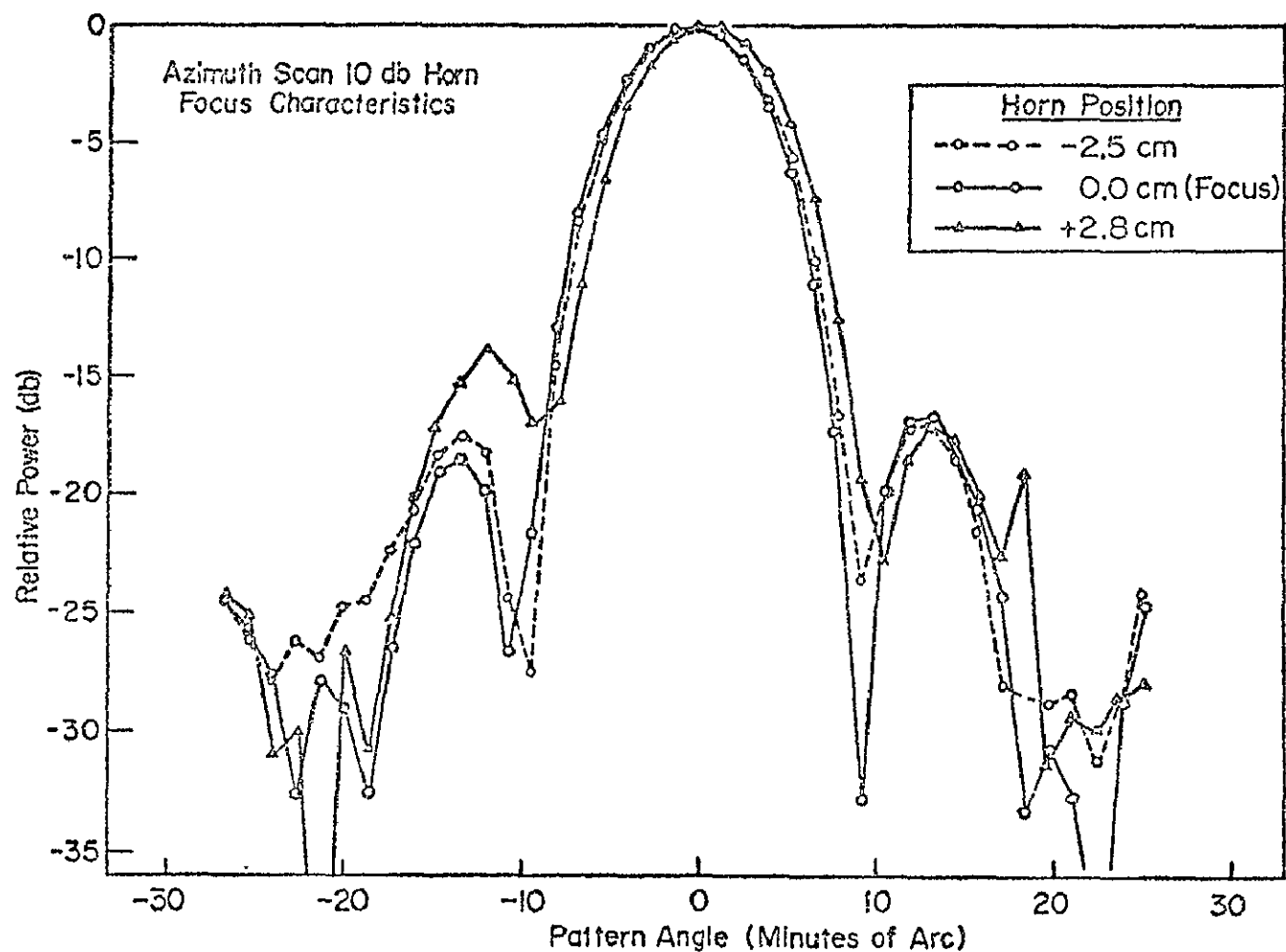


Figure B-8. Focus characteristics with the 10 dB horn. The solid line shows the antenna pattern with the horn at the focus. The two dotted lines show the pattern with the horn about one inch either side of the focus.

C.) Misalignment of the Feed Horn

If the feed horn is misaligned, such that the maximum of the horn illumination is no longer in the center of the aperture, the phase will still be constant across the aperture. If the maximum of the illumination falls at the point $(0, p_a)$ in the aperture plane ($0 \leq p \leq 1$), the approximate illumination function becomes

$$F(x,y) = E + (1-E) x [1 - (x^2 + (y - p_a)^2) / a^2]$$

Antenna patterns have been calculated using this illumination function for the dish with central blockage (no arms). Figure B-9 shows the results for various values of the horn misalignment angle α . The graph shows that for $\alpha = 1^\circ.0$ the effect is small, but it grows rapidly for $\alpha > 1^\circ.0$. This is consistent with our experimental findings.

Accuracy of Dish

Ruze (1966) reviewed the effects on the pattern of random errors in the figure of the reflector. He showed that for small errors the central part of the pattern is given approximately by the power sum of the no-error pattern and a broad pattern of scattered radiation. The ratio of the scattered radiation to the no error on axis maximum radiation is $R = \frac{1}{\eta} \left(\frac{2c}{D} \right)^2 \overline{\delta^2}$ where η is the aperture efficiency, D is the diameter of the aperture, δ is the error

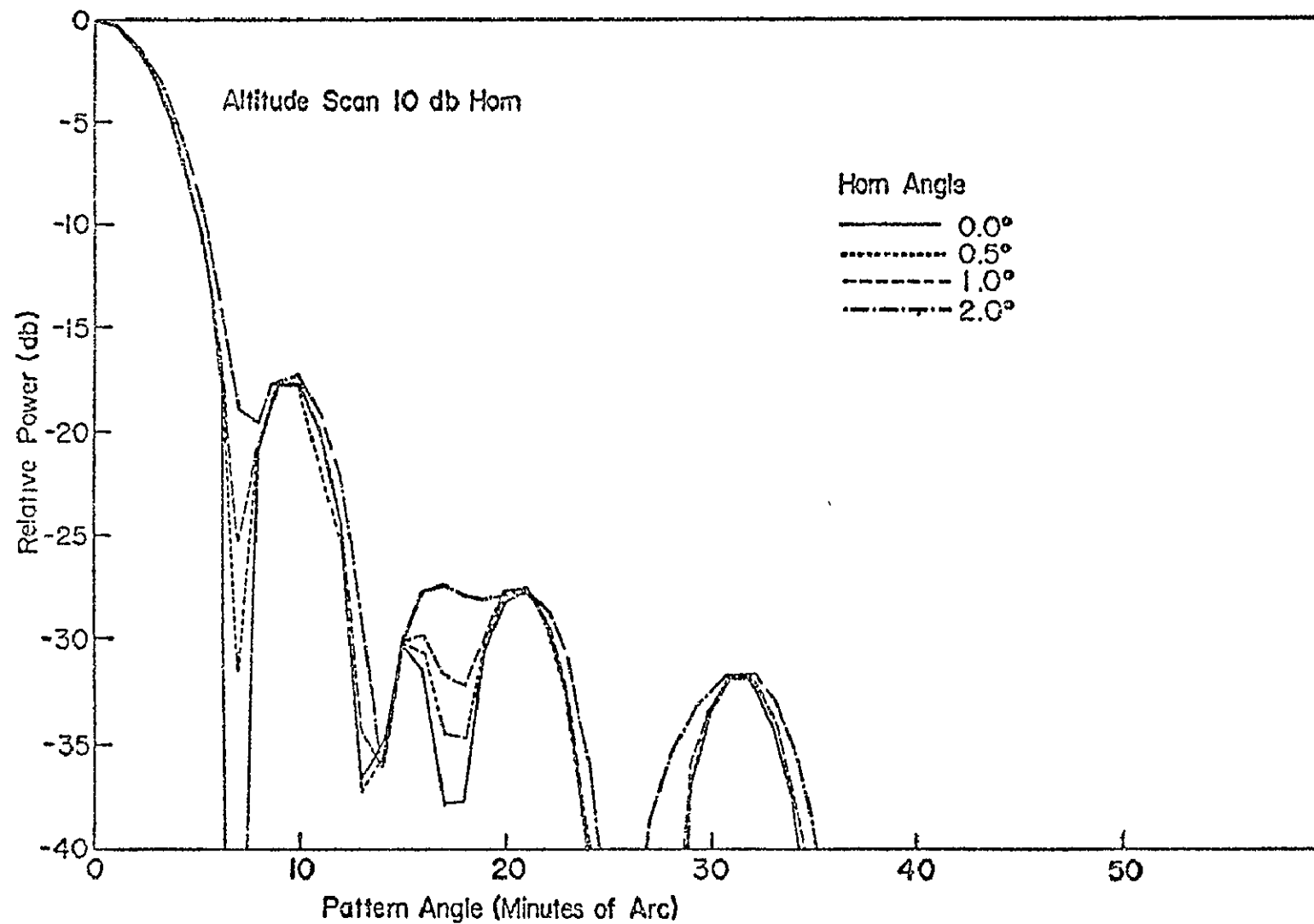


Figure B-9. Calculation of the effect of horn misalignment.

in the figure of the dish expressed in radians at the operating wavelength, and c is the typical correlation length of δ . Because the error pattern is very broad compared to the main beam, the first null in the pattern relative to the on axis maximum must be greater than R . If the measured intensity at the first null is ϵ relative to the maximum, then

$$\epsilon > (1/\eta) (2c/D)^2 \overline{\delta^2} > (2c/D)^2 \overline{\delta^2}$$

Examination of typical patterns shows that $\epsilon < 10^{-2.8}$. Thus $\delta_{rms} \equiv \sqrt{\overline{\delta^2}} \lesssim 2 \cdot 10^{-2} \frac{D}{c} \approx \frac{1}{c}$ (inches).

There may be errors of a significant fraction of a wavelength for $c \approx 1$ inch, but for large c , errors must be quite small. For example, for $c = 10$ inches, $\delta_{rms} \lesssim 1/10$ radian = $1/63$ wavelength = 0.0016 inch. For a correlation length equal to the diameter, we get $\delta_{rms} \lesssim 0.0003$ inch. This is consistent with the measured peak error of 0.0014 inch.

Conclusion

The antenna pattern has been calculated and compared with actual measurements. It was determined that both the antenna surface and the feed horn alignment were adequate to produce a diffraction limited antenna pattern that agreed well with calculation. The beamwidth was 8 arcmin full

width at half maximum and the sidelobes are more than 15 dB below the central maximum.

APPENDIX C

FITTING THE MODEL TO THE DATA

The clumpy nature of the CO spectra strongly suggests that CO is concentrated in well defined molecular clouds. We have not attempted to predict the positions of these clouds in detail, but rather, have assumed, as was explained in Chapter IV, that the clouds are randomly distributed in a way that can be described by an effective emissivity, ϵ , which is a function of galactic radius and distance above the galactic plane. As was shown, once ϵ is given, it is possible to calculate the expectation value of the antenna temperature, $\bar{T}^*$, as a function of galactic coordinates and velocity. (This is worked out in more detail in Appendix D, including the effects of beam smoothing and the finite resolution of the spectrometer.) This appendix shows how we can select a model function for ϵ which, in a sense to be described, predicts antenna temperatures which best fit the actual observed temperatures. In addition, we will discuss the accuracy of the fitted parameters.

Derivation of least-squares fitting procedure

We begin by following the usual procedure for fitting a model function to experimental data. First, using our knowledge of the physical situation, we choose a general form for $\epsilon(R,z)$ which depends on several parameters — for example, ϵ_0 , $z_{1/2}$, and z_0 in the model of Equation IV-8 — whose values are to be determined by the fitting.

From the model effective emissivity, we can then calculate, as a function of velocity and galactic coordinates, the expectation value of the antenna temperature and its dependence on the parameters. Explicitly, we write $\bar{T}^* = \bar{T}^*(X,A)$, where we abbreviate the coordinates and velocity by $X = (\ell, b, v)$ and the entire set of parameters by $A = (a_1, a_2, \dots, a_N)$, e.g. in equation IV-8 we have $A = (\epsilon_0, z_{1/2}, z_0)$ and $N = 3$.

The usual practice would now be to find the "best" value of the parameters by the least-squares procedure: by varying the parameters, A , so as to minimize the expression $\sum_{i=1}^s (\bar{T}^*(X_i, A) - T^*_i)^2$, where the experimental data are a series of s measurements of the antenna temperature, T^*_i ($i=1, \dots, s$), made at the positions and velocities $X_i = (\ell_i, b_i, v_i)$. However, this method really applies only if the statistical fluctuations in T^*_i have a

gaussian distribution. In the case at hand, the distribution of the fluctuations is unknown and may be very different from a gaussian. For example, to take a very simple case, assume that the Galaxy is filled with identical clouds, each with the same radiation temperature, T_c . The number of clouds, n_c , observed in a given spectrometer channel is then Poisson distributed. If we further assume that each observed cloud has the same beam filling factor, f , then the antenna temperature in the channel is given by $T^* = n_c T_c f$. If $n_c < 1$ then T^* is unlikely to have any value other than 0, $f T_c$, or $2 f T_c$. If we consider the variation in antenna temperature with, for example, galactic longitude, then, in contrast to $\bar{T}^*(X,A)$, which is smoothly varying with longitude, the actual observed T^*_i is a quantized function which jumps between 0 and $f T_c$. (See Figure C-1.) It is not apparent how we should fit the smooth $\bar{T}^*$ to the very different looking T^*_i .

In fact, we do not know whether the fluctuations in T^* are gaussian, poisson as in the example, or perhaps some intermediate distribution. However, we will show that the fitting procedure is similar for both the gaussian and poisson cases and that the assumption of either form leads to similar results for the fitted parameters.

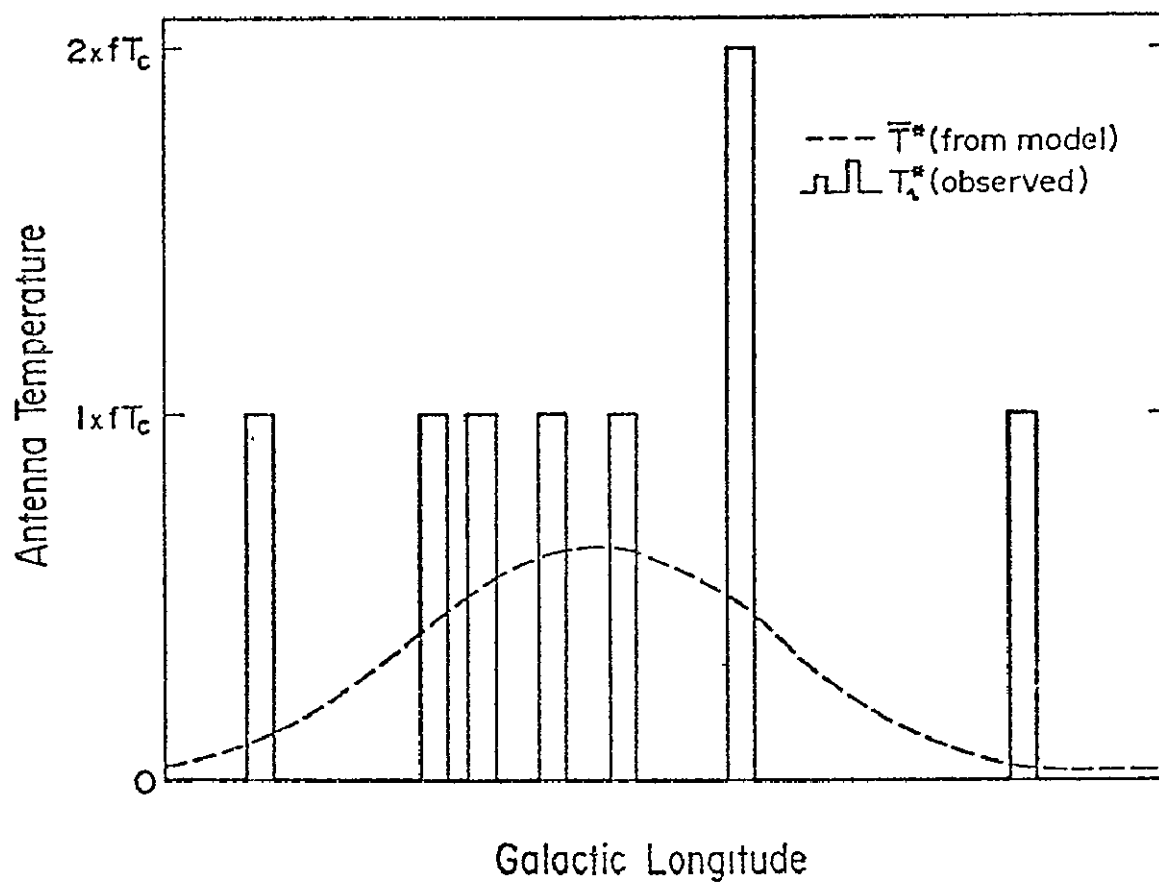


Figure C-1. Comparison of model and observed antenna temperatures for the simplified model described in the text.

Following Janossy (1965), the derivation of the procedure for least-squares fitting will be summarized briefly for both the poisson and gaussian cases. The theory of least-squares fitting is usually derived from the maximum likelihood principle. The probability of obtaining the measurement results $\underline{Y} = (y_1, \dots, y_s)$ of a series of s measurements made at the points (independent variables) $\underline{X} = (X_1, \dots, X_s)$ is given by $P(\underline{Y}; \underline{X}, A)$, where A stands for the entire set of N parameters that are to be determined, i.e., $A = (a_1, \dots, a_N)$. The best choice for A can be taken to be the value that maximizes P or, equivalently, the likelihood function, $\ln P$. If the y_i are independent random variables, we can write

$$\ln P(\underline{Y}; \underline{X}, A) = \sum_i \ln p(y_i; X_i, A) \quad (C-3)$$

where p gives the probability density for a single measurement. The best choice for A is then given by the solution of the equations

$$\frac{\partial \ln P}{\partial a_\tau} = \sum_i \frac{\partial \ln p(y_i; X_i, A)}{\partial a_\tau} = 0, \quad \tau = 1, \dots, n. \quad (C-4)$$

The variance of a_τ can be shown to be

$$\text{var}(a_\tau) = G_{\tau\tau}^{-1} \quad \text{where} \quad G_{\lambda\tau} = - \frac{\partial^2 \ln P}{\partial a_\lambda \partial a_\tau} \quad (C-5)$$

If the measurements have a Gaussian distribution,

$$p(y_i; X_i, \Lambda) = \frac{1}{\sqrt{2\pi} \sigma_i} \exp \left[- (y_i - \bar{y}_i)^2 / (2 \sigma_i^2) \right] \quad (C-6)$$

where $\bar{y}_i = \bar{y}(X_i, \Lambda)$ is the expectation value of y_i . Then,

$$\ell_n P = - \frac{1}{2} \sum_i \frac{(y_i - \bar{y}_i)^2}{\sigma_i^2} + \text{constants} \quad (C-7)$$

and the maximization of $\ell_n P$ is immediately seen to be the usual least-squares condition. Writing the equations explicitly,

$$\frac{\partial \ell_n P}{\partial a_\tau} = - \sum_i \frac{y_i - \bar{y}_i}{\sigma_i^2} \frac{\partial \bar{y}_i}{\partial a_\tau} = 0 \quad (C-8)$$

$$G_{\tau\lambda} = - \frac{\partial^2 \ell_n P}{\partial a_\tau \partial a_\lambda} = - \sum_i \frac{1}{\sigma_i^2} \frac{\partial \bar{y}_i}{\partial a_\tau} \frac{\partial \bar{y}_i}{\partial a_\lambda}, \quad (C-9)$$

For the Poisson distribution,

$$p(y_i; X_i, \Lambda) = e^{-\bar{y}_i} \bar{y}_i^{y_i} / y_i! \quad (C-10)$$

Then

$$\ln P = \sum_i (y_i \ln \bar{y}_i - \bar{y}_i) + (\text{terms not containing } A) , \quad (\text{C-11})$$

The maximum likelihood conditions are

$$\sum_i \frac{1}{\bar{y}_i} (y_i - \bar{y}_i) \frac{\partial \bar{y}_i}{\partial a_\tau} = 0 \quad (\text{C-12})$$

$$G_{\tau\lambda} = \sum_i \frac{1}{\bar{y}_i} \frac{\partial \bar{y}_i}{\partial a_\tau} \frac{\partial \bar{y}_i}{\partial a_\lambda} \quad (\text{C-13})$$

These equations are similar to Eq. (C-8) and Eq. (C-9); the only difference is that σ_i^2 is replaced with $\bar{y}_i$. For poisson statistics, $\bar{y}_i$ is in fact equal to σ_i^2 , the expected variance of $\bar{y}_i$. However, $\bar{y}_i$ is a function of A to be determined from the fit, whereas in the gaussian case σ_i^2 is a constant which must be known before the fit is made. The two sets of equations are thus actually quite different in spite of their apparent similarity.

As outlined by Janosky (p. 258) Eq. (C-12) can be solved to adequate precision by a minor modification of the usual least squares formalism. (Actually, Janosky presents a more general case.) Initial values, $A^{(0)}$, are estimated for the parameters, A . Then Eq. (C-8), the normal least squares equation, is solved with σ_1^2 held fixed at the value $\bar{y}(X_i, A^{(0)})$. From the solution new values, $A^{(1)}$, are obtained and the process is repeated with σ_i^2 now set equal to $\bar{y}(X_i, A^{(1)})$. This process is repeated until it converges.

In the usual terminology of least-squares fitting, $1/\sigma_i^2$ is called the weight of the i^{th} point; the points with the smallest uncertainties are given the largest weights. Our iterative procedure thus corresponds to a series of least-squares fits with the weights adjusted at each step to correspond to the best estimate of the uncertainties as determined from the previous step. That this procedure works is intuitively reasonable since the results of a least-squares fitting generally do not depend strongly on the weighting used.

Our fitting algorithm was checked using a stochastic simulation of the Galaxy similar to the one described in Chapter IV. The only differences were that the width of the "molecular ring" was doubled by setting $\sigma_R = 2$ kpc

(see Equation IV-9); and the number of clouds was decreased so that at the peak of the CO distribution there was only about one cloud per resolution element of the simulated spectra. The resulting spectra were fed into two versions of the least-squares fitting procedure: 1) the standard code with constant weighting — correct for gaussian statistics, and 2) the modified code — correct for poisson statistics. As can be clearly seen in Figure C- the poisson case produced considerably improved results. The error bars are a much better representation of the actual errors, and we were able to determine the parameters for two radii at which it was not possible using gaussian statistics ($R > 7.5$ kpc).

In practice, the procedure we have outlined must be modified slightly to take account of additional fluctuations due to receiver noise. To illustrate the problem we return to the example of many clouds, each with the same beam filling factor, f , and temperature, T_c . When we observe a point far from the galactic plane, the number of clouds in the beam, n_c , will be small. Thus the variance, σ^2 , of n_c , which is equal to n_c for poisson statistics, will also be small, and the weight of the point will be large. If the receiver were noise-free, this would be correct, for the detection of emission far from the

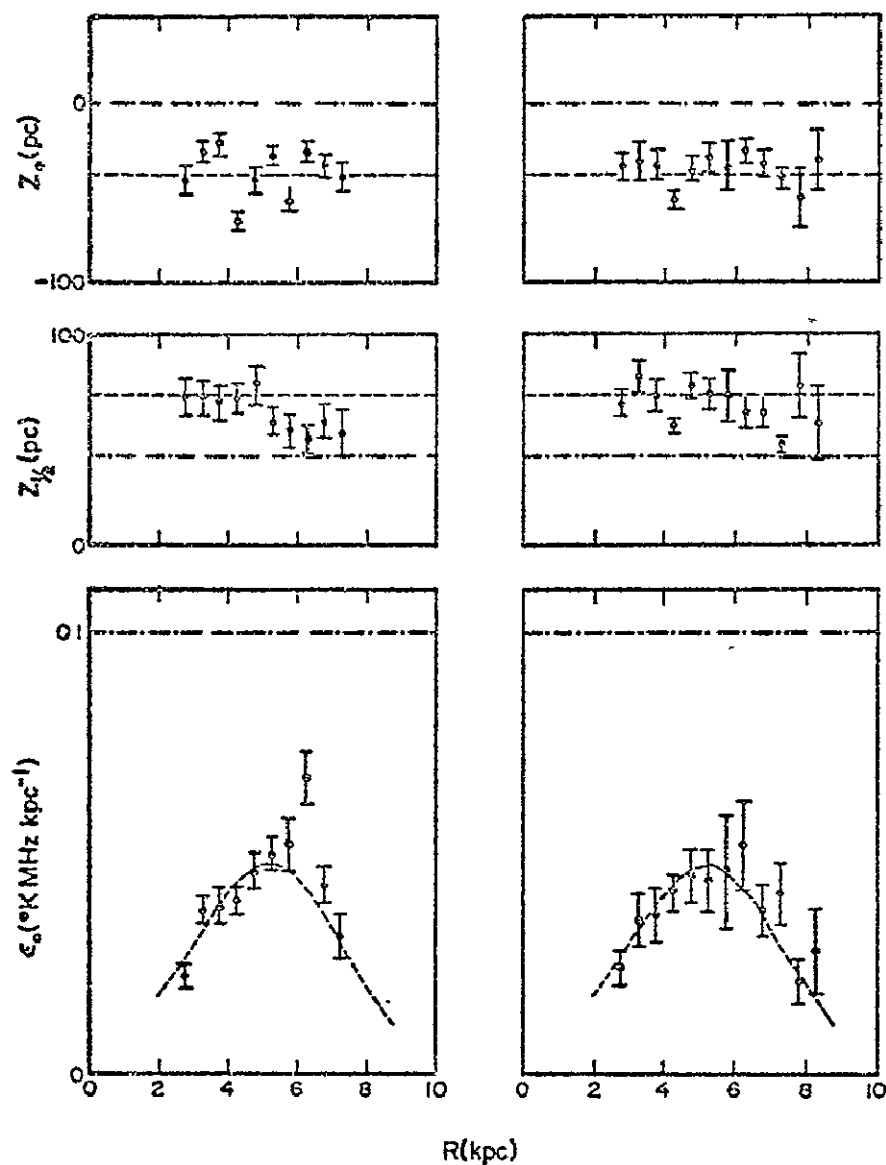


Figure C-2. Results of fitting to a simulated galaxy with a small number of clouds in each resolution element. The results on the left were obtained using the standard least-square code with constant weighting; those on the right using the modified code for poisson statistics. The dashed lines (----) indicate the underlying distribution used in simulating the galaxy. The dash-dot lines (-.-.-.-) show the original parameter estimates used in the fitting.

plane would be an extremely unlikely event. In reality, once the fluctuations in antenna temperature due to the cloud statistics fall below those due to receiver noise, increasingly larger weights should not be assigned.

To take this into account, at each step of the iteration the weights were chosen to reflect the best estimate of the fluctuations, including both receiver noise and cloud statistics; that is, the variance in the antenna temperature was taken to be

$$\sigma_i^2 = T_N^2 + \sigma_{c,i}^2 \quad (C-14)$$

where T_N is the receiver noise, and $\sigma_{c,i}^2$ is the variance due to cloud statistics. Using our simple model again, we estimate $\sigma_{c,i}^2 = (f T_c)^2 n_{c,i}$, where $f T_c$ is constant and, for poisson statistics, $n_{c,i}$ is the variance in $n_{c,i}$. If we estimate the contribution of a single cloud to the antenna temperature to be $f T_c = 1$ K, then

$$\begin{aligned} \sigma_{c,i}^2 &= (f T_c)^2 n_{c,i} \\ &= 1 \text{ K } (f T_c n_{c,i}) \\ &= (1 \text{ K}) \bar{T}^*(X_i, \Lambda^{(k-1)}) \end{aligned} \quad (C-15)$$

where we have used the best estimate of T^* from the previous iteration. So, putting the results together for the poisson case, we used the weighting function

$$1/\sigma_1^2 = [T_N^2 + (1/K) \bar{T}^*(X_1, A^{(k-1)})]^{-1} . \quad (C-16)$$

In the limit of $T_N = 0$, this is just the exact result obtained in Equation C-12.

We have actually fit the data both using the weighting as just described and also using a constant weighting function, which would be correct if the antenna temperature fluctuations were caused primarily by receiver noise. The results for these two choices, when applied to the model described in Equation IV-8 (emissivity is a gaussian in z), are shown in figure C-3. In the region of the molecular ring, both choices of the weighting give about the same results. The only significant differences appear in $z_{1/2}$ for $R > 6.5$ kpc, where the increase in thickness is somewhat less for the constant weighting.

The results given in the main body of this thesis were all obtained using constant weighting. Although this ignores the problem of counting statistics, we feel that, in view of the demonstrated insensitivity to the choice of weighting function, it is preferable because it does not introduce additional parameters, such as $f T_c$, whose values are extremely uncertain.

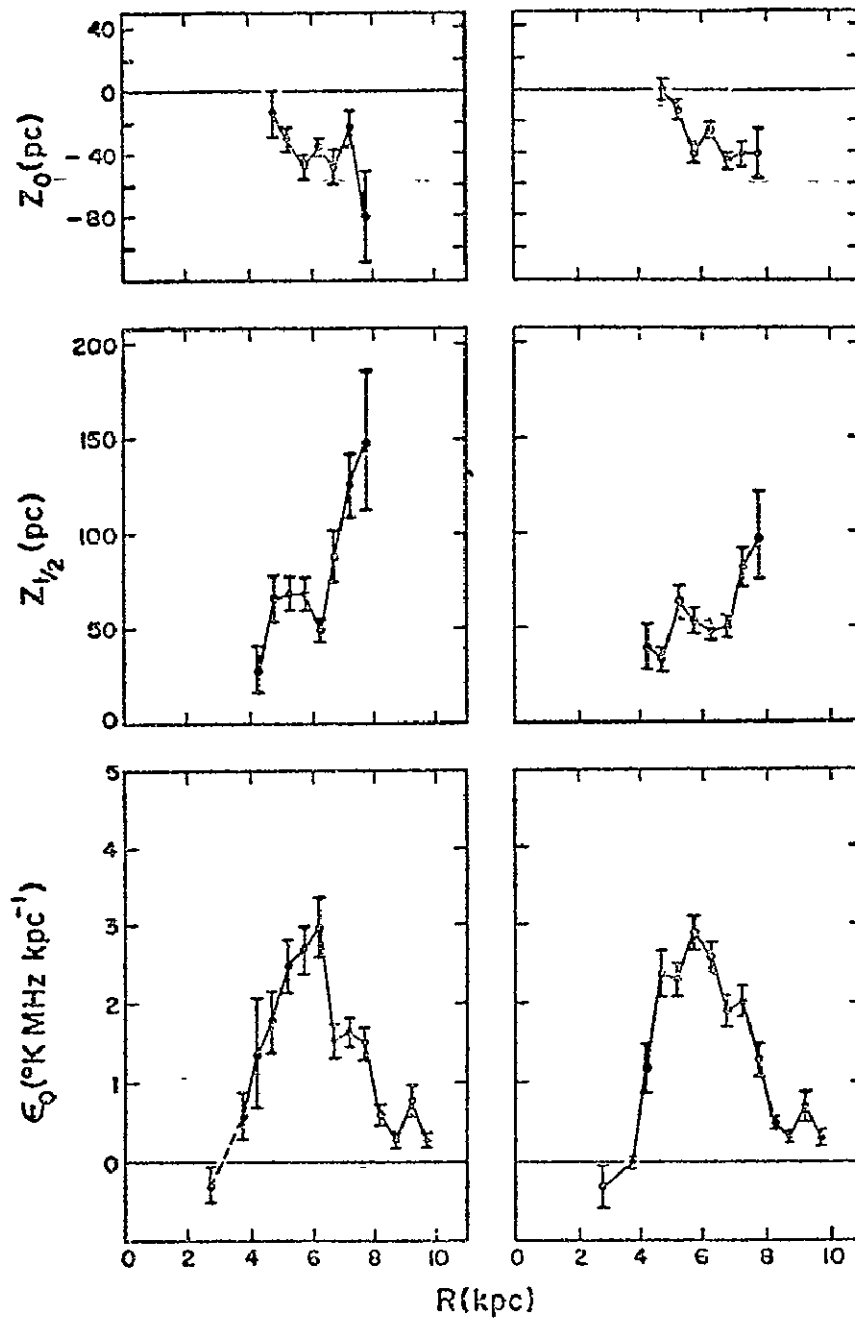


Figure C-3. Effect of choice of weighting function, σ_i^2 .
 Left: Displacement, half thickness and central emissivity of CO disk determined with $\sigma_i^2 = \overline{T}^* (X_i, A^{(k-1)}) + T_N^2$.
 Right: Same parameters determined with $\sigma_i^2 = \text{constant}$.
 See text for details.

Accuracy of the parameters

We turn now to a brief discussion of the accuracy to which the parameters can be determined. The variance of the fitted parameters could be determined using Equations C-9 with the σ_i^2 set equal to the values used in the last iteration of equation C-8. However, the variances of the parameters, unlike the values of the parameters, are sensitive to the values of the σ_i^2 , which are not accurately known. Therefore, we used instead the usual procedure for finding "internal errors" when the fluctuations in the data are not accurately known. That is, the σ_i were first scaled by a factor k so as to make the χ^2 of the data equal to its expectation value:

$$\sum_{i=1}^s \frac{1}{(k\sigma_i)^2} (T_i^* - T^*(X_i, A))^2 = s - N. \quad \text{These scaled values, } (k\sigma_i)^2, \text{ were then used in eq. C-9.}$$

The preceding discussion has assumed that the T_i^* are all uncorrelated. In fact correlations probably exist because the linewidth and spacial extent of a cloud cover more than one resolution element of the survey. To include these effects properly, additional parameters describing the properties of individual clouds would have to be introduced. Because the quantity of data now available does not seem to justify such

complex models, the correlations have been ignored. The major effect of this gross approximation will be to underestimate the errors on the fitted quantities.

The correlation can be roughly estimated by examining the spectra. A typical feature seems to cover about 3 channels (7.8 km sec^{-1}) and 2 resolution elements in latitude (0.5°) or a total of $3 \times 2 = 6$ resolution elements. The errors could, in the worst case, be underestimated by about $\sqrt{6}$. In practice, however, the error bars probably reflect systematic errors in the model and are relatively insensitive to the quantity of data. The error bars should thus be interpreted as only a rough guide to the size of the actual errors.

APPENDIX D

CALCULATION OF MODEL ANTENNA TEMPERATURES

In order to compare a model emissivity function with the survey data, it is first necessary to convert emissivity as a function of galactic position into antenna temperature as a function of telescope position and frequency. In doing this calculation several approximations have been made. These will be described in this appendix.

If, as we have assumed, the Galaxy is azimuthally symmetric, then the model specific volume emissivity (or emission coefficient) of CO can be written $\epsilon_{\tilde{\nu}}(R,z)$ where R is galactic radius, z is height above the galactic plane, and $\tilde{\nu}$ is the frequency as measured by an observer moving with the gas at location (R,z) . To evaluate the intensity at the earth it will be convenient to use the earth-centered spherical coordinates (ℓ,b,D) where ℓ and b are the usual galactic coordinates and D is distance from the earth. R and z can be expressed in these coordinates as

$$\begin{aligned} R(\ell,b,D) &= (D^2 + R_0^2 - 2R_0 D \cos b \cos \ell)^{\frac{1}{2}} \quad (D-1) \\ z(b,D) &= D \sin b \end{aligned}$$

where $R_0 = 10$ kpc is the distance from the earth to the galactic center. Because the gas is in general moving with

respect to the local standard of rest (LSR), it is useful to express $\tilde{\nu}$ in terms of the observed Doppler shifted frequency ν :

$$\tilde{\nu} = \nu(1 + V/c) \quad (D-2)$$

where V is the line-of-sight velocity of the gas with respect to the LSR. If the Galaxy is in circular rotation then (Burton, 1974)

$$V = R_O [\omega(R) - \omega(R_O) \sin \ell] \quad (D-3)$$

where $\omega(R)$ is the angular velocity of the gas. We have used Burton's (1974) formula for $\omega(R)$:

$$R \omega(R) = 250.0 + 4.05 (10-R) - 1.62 (10-R)^2 \quad (D-4)$$

The specific intensity as measured from the earth is

$$I_{\nu}(\ell, b) = \int_0^{\infty} \epsilon_{\tilde{\nu}}(R, z) dD \quad (D-5)$$

where $\tilde{\nu}$, R , and z are given by equations (D-1) and (D-2). We are interested in the intensity averaged over a single channel of the spectrometer:

$$\bar{I}_{\nu} = (1/\Delta\nu) \int_{\nu-\Delta\nu/2}^{\nu+\Delta\nu/2} I_{\nu'} d\nu' \quad (D-6)$$

where $\Delta\nu$ is the width of the channel centered at ν . Substituting equation (D-5) into equation (D-6) we get

$$\bar{I}_{\nu} = (1/\Delta\nu) \int_0^{\infty} \int_{\nu-\Delta\nu/2}^{\nu+\Delta\nu/2} \epsilon_{\tilde{\nu}'}(R, z) d\nu' dD \quad (D-7)$$

where $\tilde{\nu}'$ is derived from ν' by equation (D-2).

In the absence of any information of the line shape we assume that the line is sufficiently narrow compared to Δv that we can make the approximation

$$\varepsilon_v(R, z) = \varepsilon(R, z) \delta(v - v_0) \quad (D-8)$$

where $\varepsilon(R, z)$ is the emissivity integrated over the line and v_0 is the rest frequency of the CO transition. We can now evaluate the inner integral of equation (D-7) as

$$\int_{v-\Delta v/2}^{v+\Delta v/2} \varepsilon_{\tilde{v}'}(R, z) dv' = \begin{cases} \varepsilon(R, z), & \tilde{v}-\Delta v/2 < v_0 < \tilde{v}+\Delta v/2 \\ 0, & \text{otherwise} \end{cases} \quad (D-9)$$

Substituting into equation (D-7) we get

$$\bar{I}_v = (1/\Delta v) \int \varepsilon[R(\ell, b, D), z(b, D)] dD \quad (D-10)$$

where the integral extends over all D satisfying

$$|\tilde{v} - v_0| < \Delta v/2 \quad (D-11)$$

or, substituting for $\tilde{v}$ from equation (D-2)

$$|v[1+V(\ell, R)/c] - v_0| < \Delta v/2 \quad (D-12)$$

where $R = R(\ell, b, D)$.

As $\frac{\partial V}{\partial R} < 0$ everywhere (at least for the model rotation curve we are using), it is clear that the positions along the line-of-sight that satisfy equation (D-12) must lie between two radii $R_<$ and $R_>$. (See figure D-1). If $R_{sc} < R_<$, where $R_{sc} = R_0 \sin \ell$ is the galactic radius of the subcentral point, then in the inner part of the Galaxy (i.e., $R < R_0$) there are

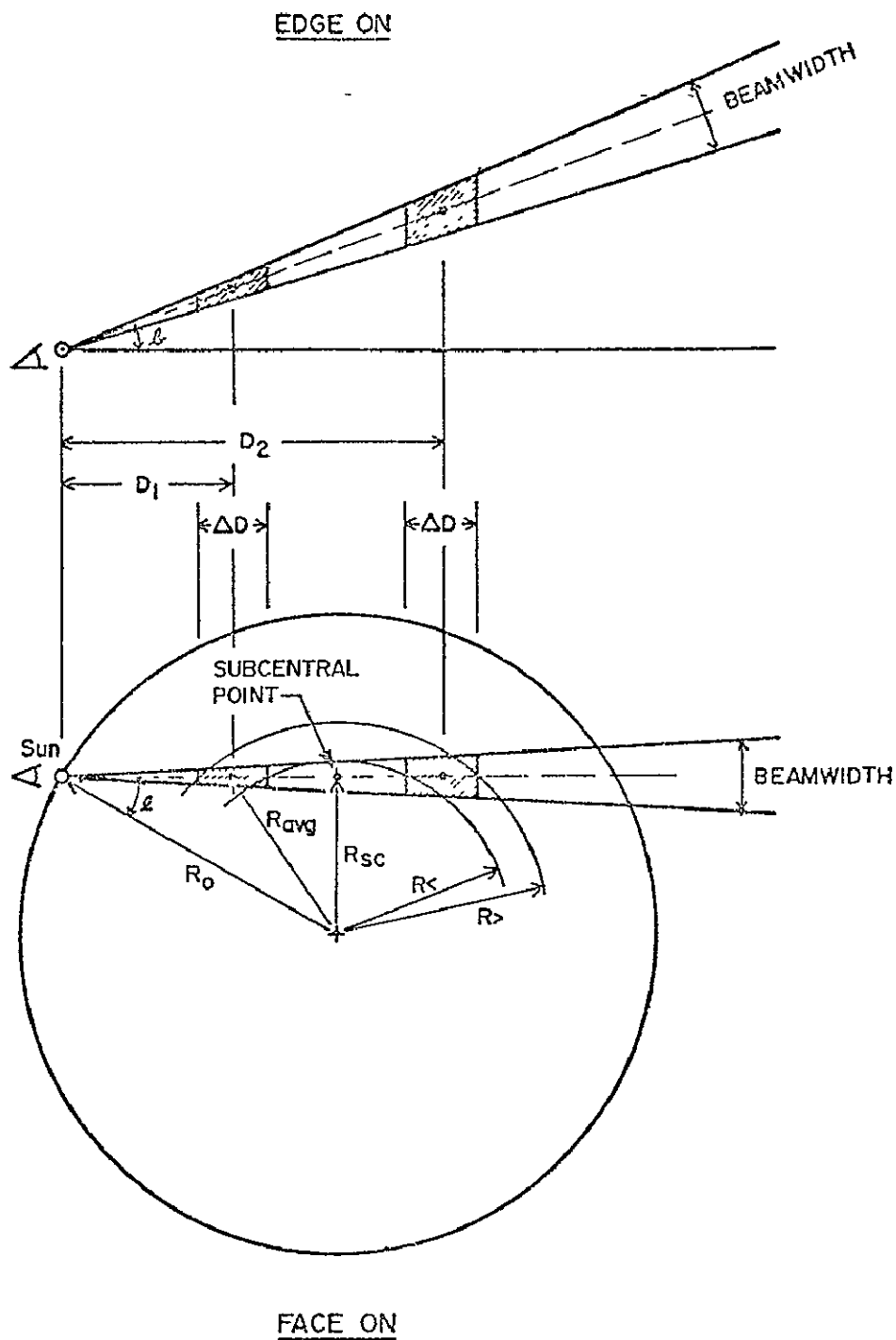


Figure D-1. Definition of coordinates in galactic plane.

two segments along the line-of-sight that satisfy equation (D-12). If $R_< < R_{sc} < R_>$, then the two segments merge. However if we redefine $R_< \equiv R_{sc}$ in this case, then it reduces to the previous one. If $R_> < R_{sc}$ then there is no solution of equation (D-12) along the line-of-sight and $\bar{I}_v = 0$. Thus we need only consider further the first case, $R_{sc} < R_<$.

We now evaluate $\bar{I}_v$ in equation (D-10). The integral can be split into two pieces, one covering each of the two segments between $R_<$ and $R_>$. In each segment the function ϵ was replaced with its value at the point along the line of sight at the radius $R_{avg} = \frac{1}{2}(R_< + R_>)$. If D_1 and D_2 are the distances to these two points and ΔD is the identical length of each segment then

$$\bar{I}_v = (\Delta D / \Delta v) \sum_{i=1}^2 \epsilon[R_{avg}(\ell, b, v), z(b, D_i)]. \quad (D-13)$$

The replacement of R by R_{avg} causes only a small error in $\bar{I}_v$ but results in an enormous simplification of the final least-squares fitting. For the spectrometer that was actually used, $R_> - R_< < 0.4$ kpc over the entire area of the survey. Thus, in substituting R_{avg} for R in equation (D-10), ϵ is evaluated at a radius less than $\frac{1}{2}(R_> - R_<) = 0.2$ kpc from where it should be. This is less than the resolution of the models we have used, and in any case larger errors result from the use of a circular rotation model. With this approximation, $\bar{I}_v(\ell, b)$, and ultimately the model antenna temperature, depend on the value of ϵ at only a single value of R . Thus, when, in the

final models, the Galaxy is split into concentric rings, the data points can be grouped according to which ring they depend upon. This lack of "mixing" between the rings allows the parameters in each ring to be least-squares fitted independently, thus vastly reducing the amount of computation needed. Finally, to get the model antenna temperature we must convolve $\bar{I}_\nu(\ell, b)$ with the beam of the antenna and multiply the result by $c^2 / (2k\nu^2)$ to convert to the usual radio astronomy units:

$$T^*(\ell, b, \nu) = \frac{1}{4\pi} \frac{c^2}{2k\nu^2} \iint_{4\pi} \bar{I}_\nu(\ell', b') G(\ell', b'; \ell, b) d\Omega' \quad (D-14)$$

where $G(\ell', b'; \ell, b)$ is the gain of the antenna (relative to an isotropic radiator) in the direction (ℓ', b') with the axis of the telescope pointed in the direction (ℓ, b) . If we define $\epsilon^* \equiv \epsilon c^2 / (2k\nu^2)$, a convenient unit of ϵ for radio astronomy, and substitute equation (D-13) into equation (D-14) we get

$$T^* = \frac{\Delta D}{4\pi\Delta\nu} \sum_{i=1}^2 \iint_{4\pi} \epsilon^* [R_{\text{avg}}(\ell', b', \nu), z(b', D_i(\ell', b', \nu))] G(\ell', b'; \ell, b) d\Omega' \quad (D-15)$$

Because the telescope beam is very small compared to the scale on which we will attempt to measure radial fluctuations in the emissivity, it is safe to evaluate R_{avg} and D_i only at the center of the beam.

The gain function of the telescope can be accurately modeled by

$$G(\ell', b'; \ell, b) \approx (4/\lambda^2) \exp[-((\ell - \ell')^2 + (b - b')^2)/\lambda^2] \quad (D-16)$$

where λ is chosen to produce a halfwidth at half maximum of 8 arc min. Since the beamwidth is much less than one radian and $b \ll 1$, we can replace the spherical integral by

$$\iint_{\pi} d\Omega \rightarrow \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\ell db \quad (D-17)$$

and also make the approximation

$$z(b, D) \approx bD. \quad (D-18)$$

The integral over ℓ' can now be done analytically with the result

$$T^* = \frac{\Delta D}{\Delta v \lambda \sqrt{\pi}} \sum_{i=1}^2 \int_{-\infty}^{\infty} \epsilon^* [R_{\text{avg}}(\ell, b, v), b' D_i(\ell, b, v)] \exp[-(b - b')^2/\lambda^2] db'. \quad (D-19)$$

In practice, we have considered two forms for the function ϵ^* -- gaussian and exponential in z . In both cases the remaining integral over b' can be done analytically. In the gaussian case we take

$$\epsilon^*(R, z) = \epsilon_0(R) \exp[-(z - z_0(R))^2/\alpha(R)^2] \quad (D-20)$$

where $\alpha(R) \equiv z_{\frac{1}{2}}(R)/(\ln 2)^{\frac{1}{2}}$

Then

$$T^* = \frac{\Delta D \epsilon_0}{\Delta v} \sum_{i=1}^2 \frac{\alpha}{(\alpha^2 + \beta_i)^{\frac{1}{2}}} \exp\left[-\frac{(z_i - z_0)^2}{\alpha^2 + \beta_i}\right] \quad (D-21)$$

where $\beta_i \equiv \lambda D_i$

$$z_i \equiv b D_i$$

In the exponential case we take

$$\epsilon^*(R, z) = \epsilon_o(R) \exp\left[-|z - z_o(R)| / \alpha(R)\right] \quad (D-22)$$

where $\alpha(R) \equiv z_{1/2} / (\ln 2)$

Then

$$T^* = \frac{\Delta D \epsilon_o}{2 \Delta v} \sum_{i=1}^2 \exp\left[\beta_i^2 / (4\alpha^2)\right] \times \quad (D-23)$$

$$\left\{ \exp\left[(z_1 - z_o)/\alpha\right] \operatorname{erfc}\left(\frac{\beta_i}{2\alpha} + \frac{z_i}{\beta_i}\right) + \exp\left[-(z_i - z_o)/\alpha\right] \operatorname{erfc}\left(\frac{\beta_i}{2\alpha} - \frac{z_i}{\beta_i}\right) \right\}$$

where β_i and z_i are the same as in eq (D-21) and erfc is the complementary error function.

Equations (D-21) and (D-23) are the actual expressions that were used in the computation.

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